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Proposition and learning of some belief function contextual correction mechanisms ☆

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A B S T R A C T

Knowledge about the quality of a source can take several forms: it may for instance relate to its truthfulness or to its relevance, and may even be uncertain. Of particular interest in this paper is that such knowledge may also be contextual; for instance the reliability of a sensor may be known to depend on the actual object observed. Various tools, called correction mechanisms, have been developed within the theory of belief functions, to take into account knowledge about the quality of a source. Yet, only a single tool is available to account for contextual knowledge about the quality of a source, and precisely about the relevance of a source. There is thus some lack of flexibility since contextual knowledge about the quality of a source does not have to be restricted to its relevance. The first aim of this paper is thus to try and enlarge the set of tools available in belief function theory to deal with contextual knowledge about source quality. This aim is achieved by (1) providing an interpretation to each one of two contextual correction mechanisms introduced initially from purely formal considerations, and (2) deriving extensions – essentially by uncovering contextual forms – of two interesting and non-contextual correction mechanisms. The second aim of this paper is related to the origin of contextual knowledge about the quality of a source: due to the lack of dedicated approaches, it is indeed not clear how to obtain such specific knowledge in practice. A sound, easy to interpret and computationally simple method is therefore provided to learn from data contextual knowledge associated with the contextual correction mechanisms studied in this paper.

1. Introduction

In today's society, a lot of information is accessible. Yet, for a piece of information to be useful, it must be interpreted with respect to the source that provides it, and in particular in the light of the quality of the source. Clearly, this is no easy task. First, the quality of a source may come in many guises: a source can for instance be biased, or even be totally irrelevant. Second, this quality may be only known with some uncertainty by the agent who has to interpret the piece of information [28].

The theory of belief functions [32,39,36] is a flexible framework to model and deal with uncertainty. Various tools have been developed within this framework to take into account uncertain knowledge about the quality of a source and to modify, or *correct* [19,28], a piece of information provided by the source according to this knowledge. The most common,

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and historically the first, of such tools is the discounting operation [32,33], which corresponds to the situation where the agent has some knowledge regarding the relevance of the source [28]. The discounting operation is central in numerous and diverse applications of belief function theory, such as classification [5] and information fusion [31,16,40] (see [27, Remarks 5 and 6] for more details on the role of discounting in these applications).

Since its inception, the discounting operation has been extended in different ways. Notably, its inverse, called de-discounting, is introduced and used in [7] to show that two well-known and apparently quite different classifiers based on belief functions, produce actually similar outputs in an important special case. This mechanism allows one to retract a discounting which is judged no longer valid or justified; it has the effect of strengthening, rather than weakening as is the case with discounting, a piece of information. It is applied successfully in a mailing address recognition system [19], where it is used in conjunction with discounting to correct outputs of postal address readers.

Another interesting extension is the correction mechanism proposed recently by Pichon et al. [28], in order to take into account knowledge about the truthfulness of a source, besides its relevance. Its interest resides in the fact that it offers a means to deal with sources that may lie, or that are biased in the case where the source is a sensor. As shown in [28], truthfulness assumptions are also quite interesting in that they can be used to reinterpret all connectives of Boolean logic, which in turn leads to generalize the unnormalized Dempster's rule [4,32] to all Boolean connectives – this rule being the pivotal and most often used combination rule in belief function theory.

Of particular interest in this paper is the fact that the quality of a source may also be contextual; for instance,[1] a thermometer is relevant to measure a temperature which falls within its range, but is typically useless if the temperature is outside of it; if we let $\mathcal{X} = \{-100\,°\mathrm{C}, \ldots, 1000\,°\mathrm{C}\}$ be the possible temperatures, then the context here is the range, which could be, e.g., $\{-38\,°\mathrm{C}, \ldots, 356\,°\mathrm{C}\}$ (range of mercury thermometers). Furthermore, such contextual quality may also be known with some uncertainty; for instance one may believe to some degree that a source is relevant for a given context.

To deal with such contextual knowledge, yet another extension of discounting is introduced by Mercier et al. [23], who consider the case where one has some knowledge about the relevance of the source, conditionally on different subsets (contexts) A of $\mathcal{X}$ such that the set $\mathcal{A}$ of these contexts forms a partition of $\mathcal{X}$, leading to an operation called *contextual discounting based on a coarsening*. Formally, contextual discounting based on a coarsening relies on the disjunctive rule of combination [9,33] and is related to the canonical decomposition of a belief function [34] as highlighted in [20]. This contextual correction mechanism was extended recently by Mercier et al. [21]: the set of contexts $\mathcal{A}$ for which one has some knowledge about the relevance of the source can be arbitrary (it no longer needs to form a partition of $\mathcal{X}$).

Contextual discounting based on a coarsening [23] and its extension uncovered in [21] are, to the best of our knowledge, the only contextual correction mechanisms that have been thoroughly studied in the literature. There is thus clearly a lack of tools to deal with contextual quality, since it does not have to be restricted to contextual relevance. As a matter of fact, Mercier et al. [20] introduce two other contextual correction mechanisms, which are quite interesting from a formal point of view: the first one, referred simply as *contextual discounting* in [20, Theorem 1][2], can be viewed as a generalization of contextual discounting based on a coarsening in that it has the same formal definition as this latter mechanism except that the set $\mathcal{A}$ that appears in its definition can be arbitrary; the second one is a dual reinforcement process to contextual discounting, which has a similar definition as contextual discounting, except that it relies on the unnormalized Dempster's rule, and it can also be linked to the canonical decomposition of a belief function. However, Mercier et al. [20] do not provide an interpretation for this latter correction mechanism, nor do they provide an interpretation for contextual discounting as shown in [21], hence the practical usefulness of these two contextual correction mechanisms remains unknown.

As a first step toward enlarging the set of tools dedicated to handling contextual quality, one may thus try and provide an interpretation to each one of Mercier et al. [20] contextual discounting and reinforcement processes. Mimicking what has been done for discounting with the introduction of contextual discounting based on a coarsening, one may also try and derive contextual forms of correction mechanisms that have already proved interesting in their non-contextual versions; in particular one may attempt to "contextualize" the two extensions of discounting recalled above that are the de-discounting operation and Pichon et al. [28] truthfulness-based correction mechanism. The first aim of this paper is to explore these different routes and to find out whether they can yield useful complements to contextual discounting based on a coarsening and its recent extension [21], with respect to the problem of handling contextual knowledge about the quality of a source. As will be seen, this exploration rests on a detailed analysis of Pichon et al. [28] truthfulness model.

In addition to the above issue of being able to take into account contextual knowledge about the quality of a source, an associated issue is the origin of such knowledge; it is indeed not totally clear how to obtain such specific knowledge in practice. Two different approaches [11,23] have been proposed to find out the contextual quality of a source, and more precisely to discover it from available labeled data. Elouedi et al. [11] approach is based on the use of confusion matrices. Its simplicity makes it quite appealing. However, it is restricted to the case of contextual discounting based on a coarsening, where the coarsening is fixed to the partition of singletons. Besides, it basically amounts to assuming that a source makes a correct prediction only when it is relevant, which is debatable (a non-relevant source may provide correct information, see, e.g., [28]). Mercier et al. [23] approach on the other hand, relies on the minimization of an error criterion. It is quite

[1] Other examples of contextual quality will be given in later sections of this paper.

[2] The operation referred to as contextual discounting in [20, Theorem 1] was thought – erroneously as shown in [21] – to be the extension to an arbitrary set of contexts, of contextual discounting based on a coarsening, hence its name. To ensure consistency with previous published works, the same name is used for this operation in this paper, although the results in [21] suggest this name may be somewhat of a misnomer.