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Causal compositional models in valuation-based systems with examples in specific theories

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ABSTRACT

We show that Pearl's causal networks can be described using causal compositional models (CCMs) in the valuation-based systems (VBS) framework. One major advantage of using the VBS framework is that as VBS is a generalization of several uncertainty theories (e.g., probability theory, a version of possibility theory where combination is the product t-norm, Spohn's epistemic belief theory, and Dempster–Shafer belief function theory), CCMs, initially described in probability theory, are now described in all uncertainty calculi that fit in the VBS framework. We describe conditioning and interventions in CCMs. Another advantage of using CCMs in the VBS framework is that both conditioning and intervention can be easily described in an elegant and unifying algebraic way for the same CCM without having to do any graphical manipulations of the causal network. We describe how conditioning and intervention can be computed for a simple example with a hidden (unobservable) variable. Also, we illustrate the algebraic results using numerical examples in some of the specific uncertainty calculi mentioned above.

1. Introduction

In many situations, we are faced with the question of what would happen if we made some changes, such as if we intervened by an action that changes the status quo. In [15], Pearl shows that such questions can be answered using *causal probabilistic models*, because of their ability *to represent and respond to external or spontaneous changes*. He also states that such questions cannot be answered with the help of non-causal probabilistic models, in which causal relations are not taken into consideration. The reason is obvious. Suppose that we have a joint probability distribution for two binary variables S and A. We can check whether they are related or not. But even if $S = s$ is strongly positively related to $A = a$, there is no way to determine that one is a cause of the other. We can only see that $P(A = a|S = s) > P(A = a)$, and $P(S = s|A = a) > P(S = s)$. However, if we are informed that $S = s$ denotes 'there is smoke in a room,' and $A = a$ denotes 'smoke alarm is sounding,' then we have more information about the situation than we had before. We now know that $S = s$ is a cause of $A = a$. In this case, we can compute not only the conditional probabilities as indicated above, but also the effects of interventions. Consider, e.g., two interventions: creating smoke in the room (e.g., by lighting a smoke bomb), and sounding the alarm (e.g., by pushing the test button on the smoke alarm). Using Pearl's *do-calculus*, we denote the first intervention by $do(S = s)$, and the second one by $do(A = a)$. Using this notation, for the effect of the first intervention,

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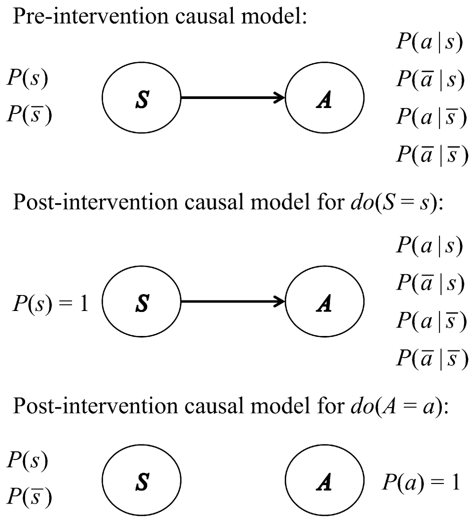

Fig. 1. Graphical modifications under interventions in the smoke-alarm causal model.

it is clear that $P(A = a|do(S = s)) = P(A = a|S = s) > P(A = a)$, because we know that $S = s$ is a cause of $A = a$. On the other hand, if we have $do(A = a)$, it activates the alarm, but we do not expect to find smoke in the room as a consequence. Clearly, $P(S = s|do(A = a)) = P(S = s)$ (see Fig. 1). In summary, when we have an intervention $do(X = x)$, where x is a state of variable X, we change the joint distribution such that all causal arcs that point to X disappear along with the conditional probability table for X (in the post–intervention model), we create a new probability distribution for X, $P(X = x) = 1$, and the remaining causal arcs and conditional probability tables remain unchanged.

In [8], causal probabilistic models, which are graphical probability models, were described by causal compositional models, which are algebraic probability models. Compositional models are based on the idea that a multidimensional probability model can be assembled, i.e., *composed*, from a collection of its lower-dimensional marginals. Naturally, it can be done only under some assumptions about the conditional independence relations between the variables for which the distribution is defined. Compositional models [7] are equivalent to Bayesian networks in the sense that a multidimensional probability distribution represented by a Bayesian network can be expressed also as a compositional model with practically the same number of parameters (probabilities), and vice-versa. The main difference stems from the fact that the building blocks of a Bayesian network are conditional probability distributions, whereas the building blocks of a compositional model are marginal distributions. These facts suggest that there are some computational advantages when computing marginals of a multidimensional model represented by a compositional model. As we will see in this paper, causal compositional models bring further advantages in comparison with causal probabilistic graphical models. The main advantage is that the effect of interventions can be described algebraically without having to do any graphical manipulations. One can compute the effect of an intervention to the original causal model without any graphical modifications, and the algebraic descriptions of conditioning and intervention differs only by a pair of parentheses. The disadvantages of causal compositional models are that these are relatively new, and consequently, not as well understood as causal graphical models.

The valuation-based system (VBS) framework was initially introduced in [20], and expanded further in [19,22,14,13,16]. VBS is an abstract framework that is able to capture many different systems such as propositional logic, uncertainty calculi, systems of equations, optimization problems, data-base systems, etc. VBS consists of objects called variables and valuations, and three operators called combination, marginalization, and removal, that operate on valuations. A valuation encodes knowledge about a subset of variables, and the subset of variables is referred to as the *domain* of the valuation. Combination allows us to combine two or more valuations resulting in a valuation that has aggregated knowledge from its constituents. In a specific VBS, if we combine all valuations in the system, we get what is called the *joint* valuation. Marginalization allows us to coarsen valuations allowing us to deduce the knowledge contained in a valuation for a variable in its domain. The coarsened valuation is called the *marginal* of the original valuation. Removal allows us to remove knowledge contained in a valuation from another valuation, and can be regarded as an inverse of combination. The three operators are required to obey some basic axioms, which enable us to compute marginals of the joint valuation for a particular variable of interest without having to explicitly compute the joint valuation. This is of interest in many systems as it may be computationally intractable to explicitly compute the joint valuation when we have many variables. The process of computing marginals of the joint valuation without explicitly computing the joint is called *local computation*.

The main contribution of this paper is a generalization of Pearl's probabilistic causal models to non-probabilistic uncertainty calculi. In [11], compositional models were described in the valuation-based system (VBS) framework, so that they are defined in all uncertainty calculi that the VBS framework is able to capture. In this paper, we extend compositional models described in the VBS framework further, so that we can represent the effects of conditioning and interventions. Thus, in doing so, we have extended Pearl's causal models to all uncertainty calculi that fit in the VBS framework. In particular,