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On some types of covering rough sets from topological points of view

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ABSTRACT

The concept of coverings is one of the fundamental concepts in topological spaces and plays a big part in the study of topological problems. This motivates the research of covering rough sets from topological points of view. From topological points of view, we can get a good insight into the essence of covering rough sets and make our discussions concise and profound. In this paper, we first construct a type of topology called the topology induced by the covering on a covering approximation space. This notion is indeed in the core of this paper. Then we use it to define the concepts of neighborhoods, closures, connected spaces, and components. Drawing on these concepts, we define several pairs of approximation operators. We not only investigate the relationships among them, but also give clear explanations of the concepts discussed in this paper. For a given covering approximation space, we can use the topology induced by the covering to investigate the topological properties of the space such as separation, connectedness, etc. Finally, a diagram is presented to show that the collection of all the lower and upper approximations considered in this paper constructs a lattice in terms of the inclusion relation $\subseteq$.

1. Introduction

Rough set theory, which was first formulated by Z. Pawlak [1] in the early 1980s, is an effective tool to deal with vagueness and incompleteness in information systems. The theory has been successfully applied to many fields, such as machine learning, knowledge acquisition, and decision analysis, etc. Lower and upper approximations, which are the core concepts in rough set theory, are defined with respect to equivalence classes. In the investigation of rough set theory, many kinds of rough sets have been formulated for practical applications, such as the tolerance relations based rough sets [2–4], the binary relations based rough sets [5,6], and the variable precision rough sets [7,8], etc. To enlarge the application scope of rough set theory, researchers have proposed many kinds of rough sets by replacing equivalence classes with coverings [9–14]. Covering rough set theory as one of the extension models of rough set theory, which was first formulated by W. Zakowski [15], has been studied extensively [16–20]. Many useful concepts have been proposed and generalized in the study of covering rough set theory. For example, the concept of complementary neighborhoods was first proposed by L. Ma [16,21] who has studied the properties and topological importance of this concept. Zhu and Wen [22] have investigated the application of the concepts of entropy and co-entropy in covering rough set theory. The concept of multi-granulation rough sets proposed by Y. Qian and C. Liu et al. [23,24] was defined by using multi-equivalence relations. Relationships between

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lower and upper approximation operators such as duality, conjugacy, and adjointness were also investigated, for instance, M. Restrepo, C. Cornelis and J. Gómez emphasized the fundamental importance of these relationships in Ref. [25]. One of the main tasks in covering rough set theory is the reduction of coverings and a lot of useful work has been done in this field [26,27]. Another important task is the construction of approximation operators which has been given much attention [16,21]. In Ref. [28], A. Tan, J. Li and G. Lin studied the relationships between covering rough set theory and formal concept analysis with respect to approximation operators. They also investigated matrix-based methods for computing set approximations and reductions of a covering decision information systems in Ref. [29]. In Refs. [30–32], a lot of useful work has been done on the classification of different rough set models.

Although topological methods have been applied to the study of covering rough sets [33,34], they have not been used as extensively as possible. As far as I know, some researchers deal with problems in covering rough set theory only using simple topological methods. They do not make use of the covering in a covering approximation space to construct topologies. We have successfully solved this problem in this paper. Drawing on topological methods, we can find some important characteristics of a covering approximation space. For example, we have proved the following conclusion: If $(U, \mathcal{C})$ is a covering approximation space and $\cap\{C : C \in \mathcal{C}\} \neq \emptyset$, then U is a connected topological space. This paper focuses on some topological structures of a covering approximation space. Given a covering approximation space, we can construct a type of topology called the topology induced by the covering. This notion is indeed in the core of this paper. We use it to define the concepts of neighborhoods, closures, connected spaces, and components. Drawing on these concepts, we define several pairs of approximation operators. We not only investigate the relationships among them, but also give clear explanations of the concepts discussed in this paper as well. We think that topological methods are useful tools for the study of covering rough set theory. From topological points of view, we can get a good insight into the essence of covering rough sets and make our discussions concise and profound.

The rest of this paper is organized as follows. In Section 2, we outline some fundamental concepts in covering rough set theory and topology. In Section 3, we introduce a topology called the topology induced by the covering on a covering approximation space. In this section, we also define some important concepts such as subspace topologies, connected spaces, etc and we also show how to judge whether a covering approximation space is connected or not. In Section 4, several pairs of lower and upper approximation operators are defined and the topological properties about them and the connections among them are also discussed in this paper. Finally, we draw a diagram to describe the relationships among all the lower and upper approximations considered in this paper. In Section 5, we present some conclusions and discussions.

2. Preliminaries

Before proceeding further, let us first recall some basic concepts and properties in rough set theory and topology.

2.1. Basic concepts and properties in rough set theory

Let U be a finite set called a universe and let R be an equivalence relation on U. The collection of all equivalence classes induced by R is denoted by U/R. It is obvious that U/R is a partition of U. If X is a subset of U, we can define the lower approximation operator $R^-(X)$ and the upper approximation operator $R^+(X)$ as follows:

$$R^-(X) = \bigcup\{Y_i \in U/R : Y_i \subseteq X\}, \quad R^+(X) = \bigcup\{Y_i \in U/R : Y_i \cap X \neq \emptyset\}.$$

The inclusion relations

$$R^-(X) \subseteq X \subseteq R^+(X)$$

hold obviously. According to Pawlak's definition, X is called a rough set if $R^-(X) \neq R^+(X)$.

Equivalence classes are indeed in the core of the definition of rough sets. As a generalization of equivalence classes, coverings can also be used to define lower and upper approximations.

Definition 1. (See [33,35].) Let U be a universe and let $\mathcal{C}$ be a collection of nonempty subsets of U. $\mathcal{C}$ is said to be a covering of U if $U = \bigcup_{C \in \mathcal{C}} C$. The ordered pair $(U, \mathcal{C})$ is called a covering approximation space.

Definition 2. (See [33,36–39].) Let $(U, \mathcal{C})$ be a covering approximation space and let x be a point of U. The neighborhood $N(x)$ of x is the intersection of all the elements of $\mathcal{C}$ containing x; that is

$$N(x) = \cap\{C \in \mathcal{C} : x \in C\},$$

and the adhesion $P_x^{\mathcal{C}}$ of x is defined by

$$P_x^{\mathcal{C}} = \{y \in U : \forall C \in \mathcal{C}(x \in C \leftrightarrow y \in C)\}.$$