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Using the WOWA operator in robust discrete optimization problems

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ABSTRACT

In this paper a class of discrete optimization problems with uncertain costs is discussed. The uncertainty is modeled by introducing a scenario set containing a finite number of cost scenarios. A probability distribution over the set of scenarios is available. In order to choose a solution the weighted OWA criterion (WOWA) is applied. This criterion allows decision makers to take into account both probabilities for scenarios and the degree of pessimism/optimism. In this paper the complexity of the considered class of discrete optimization problems is described and some exact and approximation algorithms for solving it are proposed. Applications to the selection and the assignment problems, together with results of computational tests are shown.

1. Introduction

Most practical decision making problems arise in a risky or uncertain environment, which means that an outcome of each decision is unknown and depends on a state of the world, which may occur with some positive probability. If probabilities for the states of the world are available, then each decision leads to a *lottery*, i.e. a probability distribution over the set of all possible outcomes. A decision problem can then be reduced to establishing an ordering of the set of lotteries. According to the classic expected utility theory by von Neumann and Morgenstern [27] (see also [17]), the decision maker can assign an utility to each outcome, if he accepts some simple and appealing axioms. He can then compute an expected utility of each lottery and choose a decision which leads to a lottery with the largest expected utility.

The expected utility can be seen as a weighted average of outcomes, where the weight of each outcome is just the probability of obtaining it. Thus, in the von Neumann and Morgenstern theory, the weights are independent of the outcomes and other probabilities of the lottery. However, it has been observed in human behavior that this assumption is often violated (see [6] for a deeper discussion on this topic). Many decision makers pay more attention to unfavorable outcomes and would assign larger weights to such outcomes. In such a situation the weight of each outcome depends not only on its probability, but also on its rank in the lottery. Such weights may better reflect the pessimism/optimism of decision makers. A theory of such rank dependent, transformed probabilities was introduced by Quiggin [23] (see also [25]).

In many practical situations the probabilities of scenarios are not available. We then obtain a decision problem under uncertainty. In this case, decision makers may assign subjective probabilities to scenarios [24] and compute the expected utility with respect to these subjective probabilities. However, determining the subjective probabilities may be not an easy

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task. An alternative approach is to apply some decision criteria such as the min–max, min–max regret, Hurwicz, or Laplace ones. In particular, in the Laplace criterion we apply the principle of insufficient reason and assign equal probability to each scenario. Each decision is then evaluated as the average utility of all possible outcomes. For a deeper discussion on decision making under uncertainty and description of the criteria we refer the reader to [17].

In this paper we discuss a class of discrete optimization problems, in which a finite set of feasible solutions is specified. In the deterministic case a cost of each solution is known and a decision problem consists in choosing a solution with the minimum cost. Discrete optimization problems are often represented as integer programming ones, in which the set of feasible solutions is described in compact form by a system of constraints. A class of deterministic discrete optimization problems was described, for example, in [21]. In many practical situations, the cost of each solution is unknown and depends on a state of the world which may occur with some positive probability. Each state of the world induces a cost scenario. A *scenario set* containing all possible cost scenarios is part of the input. In this paper we assume that this scenario set contains a finite number of explicitly listed scenarios. We also assume that probabilities for the scenarios are available. Notice that, under uncertainty, the principle of insufficient reason can be applied, which assigns equal probabilities to scenarios [17]. In order to choose a solution, we will apply the *Weighted Ordered Weighted Averaging* (WOWA for short) operator, proposed by Torra [26]. Given a solution, this operator allows us to define a rank-dependent weight for this solution under each scenario. This weight can be seen as a distorted scenario probability and the WOWA criterion is then a special case of the Choquet integral with respect to distorted probabilities [11]. We can evaluate each solution as a weighted average of its costs over all scenarios. The WOWA criterion contains basic criteria used in decision making under risk and uncertainty, such as the expectation (weighted mean), maximum, minimum, Hurwicz, and Laplace ones. Furthermore, if the principle of insufficient reason is applied, then WOWA becomes the OWA criterion proposed by Yager [28].

If the uncertainty is represented by a discrete uncertainty set, it is common to use the robust approach [16] to compute a solution. In this approach we assume that decision makers are risk averse and we seek a solution which minimize the cost in the worst case. This leads to applying the min–max or min–max regret criteria to choose a solution. The traditional robust approach has, however, several drawbacks. The min–max criterion is extremely conservative and it is not difficult to show examples in which it gives unreasonable solutions [17]. In particular, applying this criterion we may get a solution which is not Pareto optimal. Furthermore, the so-called *drowning effect* may also appear [7]. If the costs under some scenario are large in comparison with the costs under the remaining scenarios, then only this bad scenario is taken into account in the process of computing a solution (information connected with the remaining scenarios is ignored). Hence, in many applications a criterion which takes into account all (or at least a subset) of scenarios is required. The traditional robust approach assumes also that no probabilities are available for the scenarios, which is not always true. By using the WOWA criterion we can overcome this drawback. We can use the information connected with scenario probabilities and soften the very conservative min–max criterion. Furthermore, the WOWA criterion is consistent with the theory of rank-dependent probabilities and, in consequence, can better reflect the real attitude of decision makers towards risk. This is particularly important when decisions are not repetitious, i.e. they are implemented only once. The WOWA operator allows us to establish a link between the stochastic and robust optimization frameworks. The distorted (rank-dependent) probabilities allows us to establish a trade-off between the expected and the maximum solution costs.

In this paper we focus on the computational properties of the considered problem. Since the maximum criterion is a special case of the WOWA criterion, all negative results known for the robust min–max problems remain valid if the WOWA criterion is used. Unfortunately, the min–max versions of all basic discrete optimization problems become NP-hard even for two scenarios. This is the case for the shortest path, minimum spanning tree, minimum assignment, minimum cut, or minimum selecting items problems [16,2,4]. All these aforementioned problems become strongly NP-hard and also hard to approximate when the number of scenarios is part of the input [13,14,12]. Furthermore, when the OWA operator is used to choose a solution, then network problems (the shortest path, minimum spanning tree, minimum assignment, minimum cut) are not at all approximable [15]. However, for an important case of nonincreasing weights in the OWA operator, there exists an approximation algorithm with some guaranteed worst case ratio and the aim of this paper is to generalize this algorithm to the more general WOWA criterion. In the existing literature, the OWA operator and the more general Choquet integral have been recently applied to some multiobjective optimization problems in [10,9,8]. In these papers the authors propose some exact methods for solving the problems, which are based on a MIP formulation and a branch and bound method.

This paper is organized as follows. In Section 2, we present the problem formulation and show a motivation for using WOWA as a criterion for choosing a solution under risk and uncertainty. In Section 3, we recall some known complexity results for the considered problem. In Section 4, we propose an approximation algorithm for solving the problem, which can be applied to a large class of discrete optimization problems. Section 5 describes a method of constructing a mixed integer programming formulation, which can be used to solve the considered problem exactly. This method will be adopted from [20]. Finally, in Section 6, we show applications of the proposed model to the selection and the assignment problem. This section also contains results of computational tests, which describe the efficiency of the MIP formulation and the quality of the solutions that are returned by the approximation algorithm designed in Section 4.

2. Problem formulation

Let $E = \{e_1, \dots, e_n\}$ be a finite set of elements and let $\Phi \subseteq 2^E$ be a set of feasible solutions. In a deterministic case, each element $e_i \in E$ has a nonnegative cost c_i and we seek a feasible solution $X \in \Phi$, which minimizes the total cost