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Stream-based inconsistency measurement

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ABSTRACT

Inconsistency measures have been proposed to assess the severity of inconsistencies in knowledge bases of classical logic in a quantitative way. In general, computing the value of inconsistency is a computationally hard task as it is based on the satisfiability problem which is itself NP-complete. In this work, we address the problem of measuring inconsistency in knowledge bases that are accessed in a *stream* of propositional formulæ. That is, the formulæ of a knowledge base cannot be accessed directly but only once through processing of the stream. This work is a first step towards practicable inconsistency measurement for applications such as Linked Open Data, where huge amounts of information is distributed across the web and a direct assessment of the quality or inconsistency of this information is infeasible due to its size. Here we discuss the problem of stream-based inconsistency measurement on classical logic, in order to make use of existing measures for classical logic. However, it turns out that inconsistency measures defined on the notion of minimal inconsistent subsets are usually not apt to be used in the streaming scenario. In order to address this issue, we adapt measures defined on paraconsistent logics and also present a novel inconsistency measure based on the notion of a *hitting set*. We conduct an extensive empirical analysis on the behavior of these different inconsistency measures in the streaming scenario, in terms of runtime, accuracy, and scalability. We conclude that for two of these measures, the stream-based variant of the new inconsistency measure and the stream-based variant of the *contension* inconsistency measure, large-scale inconsistency measurement in streaming scenarios is feasible.

1. Introduction

Inconsistency measurement is a subfield of Knowledge Representation and Reasoning (KR) that is concerned with the quantitative assessment of the severity of inconsistencies in knowledge bases. Consider the following two knowledge bases $\mathcal{K}_1$ and $\mathcal{K}_2$ formalized in propositional logic:

$$\mathcal{K}_1 = \{a, b \vee c, \neg a \wedge \neg b, d\} \qquad \mathcal{K}_2 = \{a, \neg a, b, \neg b\}$$

Both knowledge bases are classically inconsistent as for $\mathcal{K}_1$ we have $\{a, \neg a \wedge \neg b\} \models \perp$ and for $\mathcal{K}_2$ we have, e.g., $\{a, \neg a\} \models \perp$. These inconsistencies render the knowledge bases useless for reasoning if one wants to use classical reasoning techniques. In order to make the knowledge bases useful again, one can either use non-monotonic/paraconsistent reasoning techniques [29, 37] or one revises the knowledge bases appropriately to make them consistent [12]. Looking at the knowledge bases $\mathcal{K}_1$ and

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$\mathcal{K}_2$ one can observe that the *severity* of their inconsistency is different. In $\mathcal{K}_1$, only two out of four formulæ (a and $\neg a \wedge \neg b$) are *participating* in making $\mathcal{K}_1$ inconsistent while for $\mathcal{K}_2$ all formulæ contribute to its inconsistency. Furthermore, for $\mathcal{K}_1$ only two propositions (a and b) are conflicting and using e.g. paraconsistent reasoning one could still infer meaningful statements about c and d. For $\mathcal{K}_2$ no such statement can be made. This leads to the assessment that $\mathcal{K}_2$ should be regarded *more* inconsistent than $\mathcal{K}_1$. Inconsistency measures can be used to quantitatively assess the inconsistency of knowledge bases and to provide a guide for how to repair them. Moreover, they can be used as an analytical tool to assess the quality of knowledge representation. For example, one simple inconsistency measure, see e.g. [10], is to take the number of *minimal inconsistent subsets* (MIs) as an indicator for the inconsistency: the more MIs a knowledge base contains, the more inconsistent it is. For $\mathcal{K}_1$ we have then 1 as its inconsistency value and for $\mathcal{K}_2$ we have 2. A lot of different approaches of inconsistency measures and postulates for inconsistency measures have been proposed, mostly for classical propositional logic [20,13–16,27,32,31,45,10,11,2,30,19], but also for classical first-order logic [8,9], description logics [26,6,38,46], default logics [7], and probabilistic and other weighted logics [5,34,25,40,41,35,33].

Inconsistencies arise easily when many experts share their knowledge in order to construct a joint knowledge base, particularly for large knowledge bases as they appear in, e.g., Semantic Web applications [39]. So far, the field of inconsistency measurement is focused on the problem on what a *reasonable* inconsistency measure is and what properties it should satisfy. In this paper, we consider the *computational* problems of inconsistency measurement, particularly with respect scenarios where the knowledge base can only be processed in a step-by-step fashion, i.e., in *streams*. More precisely, we consider a scenario where, instead of a knowledge base $\mathcal{K}$ we are faced with a stream $\mathcal{S}$ that for any point in time $i \in \mathbb{N}$ gives us a propositional formula $\phi = \mathcal{S}(i)$. The measures we are interested in update for every time step i the currently computed inconsistency value and therefore approximate the actual inconsistency value of $\bigcup_{j=1}^{i}\{\mathcal{S}(j)\}$ with the limiting case $i \to \infty$.

To address the issue of stream-based inconsistency measurement, we present a novel inconsistency measure $\mathcal{I}_{hs}$ that is inspired by the η-inconsistency measure of Knight [21] and is particularly apt to be applied to the streaming scenario. This measure bases on the notion of a *hitting set* which (in our context) is a minimal set of classical interpretations such that every formula of a knowledge base is satisfied by at least one element of the set. We then formalize the problem of stream-based inconsistency measurement, describe desirable properties of stream-based inconsistency measures by relating the problem to the classical setting of inconsistency measurement, and propose specific instantiations for stream-based inconsistency measures. We investigate the properties and the behavior of our new measures both analytically and empirically. For the latter, we conduct an extensive empirical evaluation on artificial data. Our findings show that the stream-based variant of our novel measure, as well as a measure based on paraconsistent logics are suitable in terms of runtime, accuracy, and scalability for the stream-based scenario. In summary, the contributions of this paper are as follows:

1. We present a novel inconsistency measure $\mathcal{I}_{hs}$ based on hitting sets and show how this measure relates to other measures (Section 3).
2. We formalize a theory of inconsistency measurement in streams and relate it to the classical setting of inconsistency measurement (Section 4).
3. We provide a window-based approach for applying classical inconsistency measures to the streaming case and develop specific approaches for some concrete classical measures (Section 5).
4. We conduct an extensive empirical study on the behavior of those inconsistency measures in terms of runtime, accuracy, and scalability. In particular, we show that the stream variants of $\mathcal{I}_{hs}$ and of the *contension* measure $\mathcal{I}_c$ are effective and accurate for measuring inconsistency in the streaming scenario (Section 6).

Additionally, we give necessary preliminaries for propositional logic in Section 2, provide some review of related work in Section 7 and conclude the paper in Section 8. Proofs of technical results can be found in the appendix. This paper extends and revises the previously published paper [42] by correcting and extending technical results, providing proofs, and adding further discussion.

2. Preliminaries

Let At be a propositional signature, i.e., a (finite) set of propositions (also called atoms), and let $\mathcal{L}(\mathsf{At})$ the corresponding propositional language constructed using the usual connectives $\wedge$ (*and*), $\vee$ (*or*), and $\neg$ (*negation*).

Definition 1. A knowledge base $\mathcal{K}$ is a finite set of formulæ $\mathcal{K} \subseteq \mathcal{L}(\mathsf{At})$. Let $\mathbb{K}(\mathsf{At})$ be the set of all knowledge bases.

We write $\mathbb{K}$ instead of $\mathbb{K}(\mathsf{At})$ when there is no ambiguity regarding the signature. If X is a formula or a set of formulaæ we write $\mathsf{At}(X)$ to denote the set of propositions appearing in X. Semantics to a propositional language $\mathcal{L}(\mathsf{At})$ is given by *interpretations* and an *interpretation* ω on At is a function $\omega : \mathsf{At} \to \{\mathsf{true}, \mathsf{false}\}$. Let $\mathsf{Int}(\mathsf{At})$ denote the set of all interpretations for At. An interpretation ω *satisfies* (or is a *model* of) an atom $a \in \mathsf{At}$, denoted by $\omega \models a$, if and only if $\omega(a) = \mathsf{true}$. For $\omega \in \mathsf{Int}(\mathsf{At})$ and $\phi, \phi' \in \mathcal{L}(\mathsf{At})$ we define

- $\omega \models \neg\phi$ if and only if $\omega \not\models \phi$