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Some twin approximation operators on covering approximation spaces

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ABSTRACT

Complementary neighborhood is a conception analogous to the neighborhood that we first introduced in a former paper. In this paper, we show that two different approximation operators may have a close relationship, namely, they can be defined almost in the same way except that one uses the notion of neighborhood and another uses the complementary neighborhood. We call such two approximation operators the twin approximation operators. We give some concrete examples of the twin approximation operators. Through detailed investigation on the relationship between the neighborhood and the complementary neighborhood, we further study the properties of given twin approximation operators and investigate the relationships between different twins. We also reveal the topological properties of those twin approximation operators.

1. Introduction

Since the rough set theory was developed by Pawlak in 1982 [1], it has been successfully applied to many areas. Originally, the equivalence relation is used in the rough set theory to describe the indiscernibility of elements so as to deal with the vagueness and uncertainty in information systems. However, in real-world applications, the relations between objects are often much more complicated than the equivalence relations, and vast quantities of important information such as degrees of inclusion relations between sets and the extent of overlap of sets, etc., were not taken into account in the equivalence-relation-based rough set theory. In order to solve more and more complicated problems, researchers generalized the equivalence-relation-based rough set theory to the non-equivalent-relation-based rough set theory [2–5] covering rough set theory [6–8] and fuzzy rough set theory [9,10], etc.

As the concept of neighborhood has many practical applications in feature selection, granular computing and attribute reduction, etc. [11–15], the neighborhood-related rough sets were studied. In the non-equivalent-relation-based rough set theory, the successor neighborhood and predecessor neighborhood are two important concepts [4,16], and the k-step neighborhood system was also applied to this theory [17]. In the covering rough set theory, the concept of neighborhood induced by covering plays an important role [7,18–24]. The neighborhood-based covering rough set theory has proven to be useful in the discovery of decision rules from the incomplete information systems [14] and the attribute reduction from nominal data [12]. In Ref. [21], we first introduced the concept of complementary neighborhood in the investigation of covering rough sets.

During the investigation of covering rough set theory, various neighborhood-based lower and upper approximation operators have been defined and studied [4,23–26]. It is often difficult to find the relationship between different approximation

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operators. In this paper, based on the notions of neighborhood and complementary neighborhood, we simplified the definitions of some known approximation operators, and find that two different approximation operators may have a close relationship, namely, they can be defined almost in the same way except that one uses the notion of neighborhood and another uses the complementary neighborhood. We call such two approximation operators the twin approximation operators. We also find corresponding twin operators for some known approximation operators. Moreover, through detailed investigation of the relationship between the neighborhood and complementary neighborhood, we further reveal the properties of given twin operators.

Topology provides many valid mathematical methods and skills for the study of rough set theory, and many researchers have investigated the rough set theory from topological viewpoint [7,21,27–31]. In this paper, we also inquire into topological properties of all involved approximation operators. It is found that the twin approximation operators have similar properties.

The rest of this paper is organized as follows. In Section 2, we discuss the relationships between the neighborhood and complementary neighborhood. In Section 3, we show several pairs of twin approximations, and study the properties of them in the next section. In Section 5, we thoroughly investigate the topological properties of all those approximation operators. We conclude in the last section.

2. Neighborhood and complementary neighborhood

In this section, we recall some fundamental concepts in rough set theory and discuss the relationship between neighborhood and complementary neighborhood.

Pawlak's rough sets are based on equivalence relations, or equivalently, partitions.

Let U be a finite set called universe, and R be an equivalence relation on U. U/R denotes the family of all equivalence classes induced by R. Obviously U/R is a partition of U. For any $X \subseteq U$, the lower and upper approximations of X are defined as follows:

$$R_*(X) = \bigcup\{Y_i \in U/R : Y_i \subseteq X\}, \qquad R^*(X) = \bigcup\{Y_i \in U/R : Y_i \cap X \neq \emptyset\}.$$

According to Pawlak's definition, X is called a rough set if and only if $R_*(X) \neq R^*(X)$.

Covering is an extension to partition. Pawlak's rough set model was also extended to covering based rough sets.

Definition 1. (See [7].) Let U be a universe and $\mathcal{C}$ be a family of subsets of U. If no element in $\mathcal{C}$ is empty and $U = \bigcup_{C \in \mathcal{C}} C$, then $\mathcal{C}$ is called a covering of U, and the ordered pair $(U, \mathcal{C})$ is called a covering approximation space.

Following the sense of Pawlak, a set $X \subset U$ is called a covering rough set if its covering-induced lower approximation and upper approximation are not equal.

The concept of neighborhood plays an important role in defining approximations of sets in covering approximation spaces. In our recent paper [10], we gave a new notion of complementary neighborhood, which is analogous to neighborhood in practical and theoretical investigations of covering rough sets. It is extremely important to introduce the concept of complementary neighborhood. Based on the concepts of neighborhood and complementary neighborhood, not only can we simplify the forms of some known types of lower and upper approximation operators, but can make the relationships between some approximation operators clear and, furthermore, can define some new types of lower and upper approximation operators so as to select suitable approximations in practice. In the following, we review these two concepts, and discuss their properties and relationships.

Definition 2. (See [7,8].) Let $(U, \mathcal{C})$ be a covering approximation space. We define the neighborhood of an element $x \in U$ as

$$N(x) = \bigcap\{C \in \mathcal{C} : x \in C\}.$$

In this paper, we use $-X$ to denote the subset $U - X$, where U is the universe and $X \subset U$.

We next define the concept of complementary neighborhood in a new way which is equivalent to Definition 9 in paper [21].

Definition 3. Let $(U, \mathcal{C})$ be a covering approximation space and $x \in U$. We call

$$M(x) = -\left(\bigcup\{C \in \mathcal{C} : x \notin C\}\right)$$

the complementary neighborhood of x.

We can easily find that this definition is equivalent to

$$M(x) = \bigcap\big\{-C : (C \in \mathcal{C}) \wedge (x \notin C)\big\},$$

where $M(x) = U$ if $x \in C$ for each $C \in \mathcal{C}$.