International Journal of Approximate Reasoning 56 (2015) 137–151

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

On truth-gaps, truth-gluts, and bipolar propositions

Yongchuan Tang [a,*], Jonathan Lawry [b]

[a] College of Computer Science, Zhejiang University, Hangzhou, 310027, PR China
[b] Department of Engineering Mathematics, University of Bristol, Bristol, BS8 1TR, United Kingdom

ARTICLE INFO

Article history:
Received 10 April 2014
Received in revised form 19 August 2014
Accepted 25 August 2014
Available online 29 August 2014

Keywords:
Bipolar belief measures
Bipolar valuations
Mass function
Semantic uncertainty
Belnap four-valued logic
Kleene's strong three-valued logic

ABSTRACT

This paper assumes that each proposition can be defined by a positive criterion together with a negative criterion. Then, within the framework of propositional logic, this paper proposes Belnap valuations as a truth model for bipolar propositions. The truth value of bipolar proposition is meant to be one of four values: *absolutely true*, *absolutely false*, *borderline*, and *inconsistent*. The borderline and inconsistent cases represent the truth-gap and truth-glut of a bipolar proposition, respectively. In order to reduce inconsistency two truth normalization methods, strong truth normalization and Kleene truth normalization, are proposed to approximate the original Belnap valuations. By integrating uncertainty of the true interpretations of bipolar propositions this paper introduces the following bipolar belief measures: positive and negative belief measures, lower and upper belief measures, and Kleene lower and upper belief measures. A mass function characterization of these bipolar belief measures are further explored. The relationship with Atanassov's intuitionistic fuzzy sets and interval fuzzy sets is also discussed in detail.

1. Introduction

In natural language use, we as communicating agents constantly face decision making about the assertability or the applicability of propositions. As the rational agents we should select the most appropriate propositions according to the labeling conventions which govern the assertability of propositions. The importance of assertability in language has been stressed by many studies such as Parikh [1], Giles [2,3] and Kyburg [4]. As stated in [5] one basic argument is that the vagueness inherent in language means that predicates lack a clear and explicit definition and we therefore tend to learn the 'usage of these words in some few cases and then we extrapolate'.

We argue that the assertability of propositional expressions usually manifests itself in different bipolar ways. One popular bipolar model of assertability is that of *definite assertability* and *acceptable assertability* [5,6]. This kind of bipolarity makes a distinction between those expressions which labeling convention would deem clearly assertable, and those which labeling convention would not classify as incorrect, or perhaps even dishonest, to assert. Parikh [1] observed that:

> Certain sentences are assertible in the sense that we might ourselves assert them and other cases of sentences which are non-assertible in the sense that we ourselves (and many others) would reproach someone who used them. But there will also be the intermediate kind of sentences, where we might allow their use.

* Corresponding author. Tel.: +86 0571 87953695; fax: +86 0571 87951250.
E-mail addresses: tyongchuan@gmail.com (Y. Tang), j.Lawry@bris.ac.uk (J. Lawry).

http://dx.doi.org/10.1016/j.ijar.2014.08.006

The essence of this kind of bipolarity is that there exist borderline expressions which are *neither definitely assertable nor definitely non-assertable*. This kind of bipolar assertability can be modeled based on different truth models. For example, as pointed out in [5–7] Kleene strong three-valued logic [8] can be used. In our recent work [9] we also showed that it can be modeled on the basis of supervaluationist principles. Furthermore, this bipolarity is a special case of what Dubois and Prade [10] refer to as *symmetric bivariate unipolarity*, whereby judgements are made according to two distinct evaluations on unipolar scales. In other words, we have a strong and a weak evaluation criteria where the former corresponds to the definite assertability and the latter to the acceptable assertability. For example, consider a witness in a court of law describing a suspect as being *tall*. Depending on the actual height of the suspect this statement may be deemed as clearly true or clearly false, in which latter case the witness could be accused of perjury. However, there will also be an intermediate height range for which, while there may be doubt and differing opinions concerning the use of the description *tall*, it would not be deemed as definitely inappropriate and hence the witness would not be viewed as committing perjury.

In this paper we further explore the truth model of bipolar propositions from the viewpoint of *positive assertability* and *negative assertability*. In this context the propositional expressions are asserted according to a positive and a negative evaluation criteria. A propositional expression is positively assertable if it satisfies the positive evaluation criterion, and it is negatively assertable if it satisfies the negative evaluation criterion. As a rational agent we clearly distinguish the arguments in favor and against the labeling decisions. In this bipolar labeling decision, we are actually modeling the *truth* and *falsity* of bipolar propositions independently. From this truth model, we can explore not only the vagueness, but also the inconsistency, of propositions.

One motivation for this truth model, which deals with the *truth* and *falsity* of bipolar propositions independently, is that we are learning concept usages in a distributive and inductive language environment where the positive and negative cases of any concepts coexist. From the positive cases we may induce a positive labeling criterion and from the negative cases we may induce a negative labeling criterion. In this inductive learning process we could not expect that the positive and negative labeling criteria are completely consistent. In other words, when we label some new cases we may face the situation of truth-gluts and truth-gaps. This means that it may happen that both of positive and negative labeling criteria are able to be applied, and it may also happen that neither of these positive and negative labeling criteria are able to be applied. In machine learning for classification we also face the same problem where, however, there has been little exploration of truth models for inductive learning.

In this paper, we propose a truth model to deal with the co-existence of uncertain truth-gluts and truth-gaps. In particular, we adopt bipolar valuations (v^+, v^-) to model the labeling conventions governing the assertability of bipolar propositions, where $v^+(p) = 1$ (resp. 0) means that the positive criterion for defining proposition p is satisfied (resp. not satisfied) and $v^-(p) = 1$ (resp. 0) means that the negative criterion for defining proposition p is satisfied (resp. not satisfied). However, due to the distributive and inductive language learning environment we could not expect that the labeling conventions are certain and clearly defined. In other words, we are often uncertain about the *truth* and *falsity* of propositions due to a lack knowledge of the positive and negative evaluation criteria for defining propositions. Our perspective on bipolar propositions is referred to as *Epistemic stance* [11], which is stated concisely as follows:

> Each individual agent in the population assumes the existence of a correct set of language conventions, governing what can truthfully or falsely be asserted given a particular state of the world.

By the *epistemic stance* we are able to assume that there exists a subjective probability to model the epistemic uncertainty of labeling conventions. In other words, each bipolar valuation (v^+, v^-) is assigned a probability value $w(v^+, v^-)$. Hence it is possible to define a positive belief measure to quantify the probability that the positive criterion of the proposition is satisfied, and similarly a negative belief measure to quantify the probability that the negative criterion of the proposition is satisfied. From bipolar valuations we can also define two normalization to reduce the truth-gluts of bipolar propositions. One normalization is strong truth normalization, and the other is the Kleene truth normalization. Then bounded belief measures can be defined based on the nature of the epistemic uncertainty of the normalized bipolar valuations.

This paper is organized as follows: Section 2 introduces bipolar valuations as a general truth model for propositions, and focuses on one specific truth model: Belnap valuations. This truth model deals with *truth* and *falsity* independently. Section 3 introduces two more truth models to normalize the truth-gluts of bipolar propositions. Section 4 introduces three kinds of bipolar belief measures to quantify the semantic uncertainty of propositions within these three truth models. Section 5 explores a mass function characterization of bipolar belief measures. Section 6 presents a different perspective on Atanassov's intuitionistic fuzzy sets based on the truth model of Belnap valuations. Finally Section 7 gives some conclusions and indicates possible directions for future work.

2. Truth models of truth-gaps and truth-gluts

In the above discussions, we assume that a proposition is *true* (resp. *not true*) means that the positive criterion for defining proposition is satisfied (resp. not satisfied), and a proposition is *false* (resp. *not false*) means that the negative criterion for defining proposition is satisfied (resp. not satisfied). In this section we introduce the concept of bipolar valuation to model the truth of propositions by taking into account the *truth* and *falsity* of propositions independently. We firstly introduce the general definition of bipolar valuations, then focus on a truth model: Belnap valuations.