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Lattice-valued simulations for quantitative transition systems

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ABSTRACT

Quantitative (bi)simulations taking values from non-negative real numbers enjoy numerous applications in the analysis of labeled transition systems, whose transitions, states, or labels contain quantitative information. To investigate the simulation semantics of labeled transition systems in the residuated lattice-valued logic setting, we introduce an extension of labeled transition systems, called the quantitative transition systems (QTSs), whose labels are equipped with a residuated lattice-valued equality relation. We then establish a lattice-valued relation between states of a QTS, called approximate similarity, to quantify to what extent one state is simulated by another. One main contribution of this paper is to show that unlike the classic setting where similarity has both fixed point and logical characterizations, these results do not hold for approximate similarity on QTSs in general, but they hold for QTSs having truth values from finite Heyting algebras.

1. Introduction

Labeled transition systems (LTSs) are typically used as behavioral models of concurrent and distributed systems. A variety of preorder and equivalence relations have been proposed to capture the behaviors of LTSs from different aspects. Among them, (bi)simulations [1,24,25,34] enjoy numerous applications in the analysis of LTSs.

As pointed out in [2,9–11,16,17,36], (bi)similarity verification techniques are *qualitative* and inherently fragile: for example, two systems either are, or are not bisimulation-equivalent, regardless of how close the two systems are. To overcome this limitation, a great variety of LTSs, whose transitions, states or labels contain *quantitative* data, have been proposed to model the quantitative properties of systems. Classical similarity and bisimilarity relations have been adapted for these systems. One is based on the notion of metric or pseudometric, which assigns a distance, a non-negative real number, to each pair of states of a system (e.g., [2,11,40,41]). Such a metric yields a quantitative analogue to (bi)similarity in that the distance between states expresses the similarity of the behaviors of the states. The other approach is to introduce various approximate (bi)simulations (e.g., [16,17,39]), which characterize almost (not necessarily exact) (bi)similar states by a bound ϵ. Recently, the relationships between these two approaches have also been discovered in [18,36,40,41].

Although there are some attempts to extend (bi)simulation to the quantitative cases, most of them mainly focus on *numerical* (bi)simulation semantics of LTSs, where the truth values come from *real numbers*. It means that all the truth values are *comparable* in the view of partially ordered sets. Clearly, numerical (bi)simulation semantics has limited practically applicability. Let us consider a simple example.

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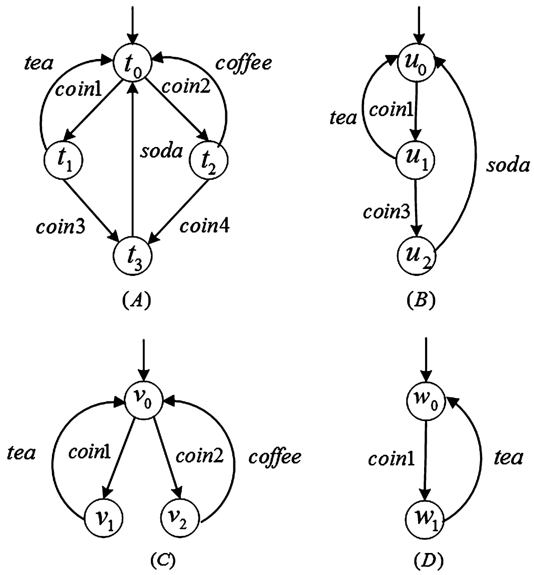

Fig. 1. Four vending machines.

Example 1.1. Implementation verification requires a model of system and specification, as well as a procedure for checking whether the two are related with respect to the given preorder or equivalence, say, simulation and bisimulation. Informally, a system simulates another system if the former's behavior pattern is at least as rich as that of the latter. It is generally accepted that if one system $\mathcal{A}$ is simulated by another system $\mathcal{B}$, then $\mathcal{A}$ is a (correct) implementation of $\mathcal{B}$. Often, two systems can be both regarded as the implementations of a specification, but the two are not simulated by each other. Consider the four vending machines in Fig. 1. For machine A, after putting in *coin1*, you can get tea by pressing the button *tea* or you put in another *coin3* and then get soda by pressing *soda*, and so on. Similarly for other machines. Clearly, all of machines B, C and D are implementations of A from the view of simulation, but B is not regarded as an implementation of C and vice versa. We obtain the following preorder relation from the view of simulation:

$$\mathcal{R} = \{(A, A), (B, B), (C, C), (D, D), (B, A), (C, A), (D, A), (D, B), (D, C)\}.$$

However, when the simulation is generalized to the numerical setting, the preorder relation becomes linear order. Hence, such a method results in the loss of the important incomparable information of preorder.

The above example shows us that it is possible that the truth values we manipulate are not numbers and are not *linearly ordered*. Thus, a study on a weaker yet rich structure has its interest. The goal of this paper is to extend the notion of simulation to the *residuated lattice-valued setting*. Residuated lattices [7,19] are a very general algebraic structure with very important applications in different areas like fuzzy automata [12,13,31,32], fuzzy logics [28,29], modal logics [5,19], and other areas [14,22,26,27,33,38]. The main challenge in this work is to make a compromise between the suitability of models as well as similarity and the preservation of lattice-valued analogues to boolean properties of similarity. We present an extension of LTSs called *quantitative transition systems* (QTSs), where the set of labels is equipped with a *residuated lattice-valued equality relation*. Such a relation is a lattice-valued version of classical equivalence relation.

Given a lattice-valued equality relation on labels, a simulation called δ-simulation is defined in the paper. The relation relaxes the equality of labels of transitions in the classical simulation by allowing to perform transitions induced by different labels, as long as the truth degree between the labels is greater than or equal to the threshold δ taken from a certain lattice. Based on the notion of δ-simulation, we introduce a lattice-valued relation between states, called approximate similarity, to quantify to what extent one state is simulated by the other. Roughly speaking, the degree that one state s_1 is simulated by another state s_2 is defined as the largest threshold δ with which s_2 approximately simulates s_1. We then show that our approximate similarity has some appealing properties which are necessary for quantitative verification: (1) It is a lattice-valued preorder relation, and thus it is applicable to some quantitative implementation verifications; (2) it gives a lower bound of lattice-valued trace inclusion relation; (3) the problem of computing the approximate similarity is P-complete.

Similarity is an elegant concept which can be described from many different perspectives. For example, it can be characterized from the view of fixed point theory and modal logic. Naturally, we want to know whether our similarity has the corresponding results. In fact, there are corresponding characterizations for QTSs when their truth structure is a finite Heyting algebra [7,19]. It has been shown that some characterizations do not hold when the truth values of a QTS come from an arbitrary residuated lattice such as standard Lukasiewicz algebra [7,19].

The present work is a continuation of [26], where Pan et al. introduced an extension of doubly labeled transition systems in the framework of complete residuated lattices, called lattice-valued doubly labeled transition systems (LDLTSs). In [26],