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On an optimization representation of decision-theoretic rough set model

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ABSTRACT

Decision-theoretic rough set model can derive several probabilistic rough set models by providing proper cost functions. Learning cost functions from data automatically is the key to improving the applicability of decision-theoretic rough set model. Many region-related attribute reductions are not appropriate for probabilistic rough set models as the monotonic property of regions does not always hold. In this paper, we propose an optimization representation of decision-theoretic rough set model. An optimization problem is proposed by considering the minimization of the decision cost. Two significant inferences can be drawn from the solution of the optimization problem. Firstly, cost functions and thresholds used in decision-theoretic rough set model can be learned from the given data automatically. An adaptive learning algorithm and a genetic algorithm are designed. Secondly, a minimum cost attribute reduction can be defined. The attribute reduction is interpreted as finding the minimal attribute set to make the decision cost minimum. A heuristic approach and a particle swarm optimization approach are also proposed. The optimization representation can bring some new insights into the research on decision-theoretic rough set model.

1. Introduction

As a kind of probabilistic rough set model, decision-theoretic rough set model (DTRS) [35–38,44] can derive current several probabilistic rough set models when proper cost functions are used, such as 0.5 probabilistic rough set model [25,27], variable precision rough set model [53] and Bayesian rough set models [30,42]. For decision-theoretic rough set model, its an important contribution to the rough set theory is that it provides a theoretic framework for calculating the thresholds required in probabilistic rough set models.

Current studies on decision-theoretic rough set model can be divided into two groups. One group concentrated on the intension and the extension of the model. Yao [37,38] investigated how to derive other probabilistic rough set models from decision-theoretic rough set model. Lingras et al. [15], Liu et al. [17–20] and Zhou [51] studied the multiple-category decision-theoretic rough set model from different viewpoint, respectively. Zhou and Li [52,12] proposed a multi-view decision model based on decision-theoretic rough set model. Users could make optimistic decision, pessimistic decision, and equable decision by adopting different values on the costs. For the attribute reduction in decision-theoretic rough set model, Yao and Zhao [39] and Zhao et al. [48] defined a general attribute reduction and analyzed several evaluation criteria. Li et al. [11] also made a further investigation on the monotonicity property of attribute reduction in decision-theoretic rough set model. Yao and Zhou [42] introduced a Naive Bayesian decision-theoretic rough set model by using Bayes's theorem to estimate the condition probabilities of objects. Yao and his colleagues [1,3] proposed a game-theoretic rough set model by implying game theory into decision-theoretic rough set model. Qian et al. [29] studied the multigranulation decision-theoretic rough set models.

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The other group concentrated on the application of decision-theoretic rough set model. Li et al. [13] proposed an instance-centric hierarchical classification framework based on decision-theoretic rough set model, and applied it to the text classification problem. Both Lingras et al. [16] and Yu et al. [45,46] applied decision-theoretic rough set model into the clustering problem. As filtering spam email is a typical three-way decision problem, many authors tried to solve it by adopting decision-theoretic rough set model. Zhao et al. [49] introduced decision-theoretic rough set model to filtering spam email problem first, and three-way decisions corresponds to three kinds of emails. Zhou et al. [50] proposed a practical approach to the filtering problem by combining Naive Baysian classifier and decision-theoretic rough set model. Jia et al. [8] integrated several classifiers into the three-way decisions framework and studied the efficiency of three-way decisions approach to filtering spam email.

Based on Bayesian decision procedure, decision-theoretic rough set model provides systematic methods for deriving the required thresholds on probabilities for defining the three regions: positive region, boundary region and negative region. For the semantics interpretation of the three regions, Yao [40,41] proposed a three-way decisions framework which consists of positive, boundary and negative rules. In decision-theoretic rough set model, all decisions are made on the basis of minimizing expected cost. The expected cost, which also be called decision cost, is a kind of classification cost and it is a core concept in decision-theoretic rough set model. In this paper, we propose an optimization representation of decision-theoretic rough set model. An optimization problem can be constructed with the objective of minimizing the decision cost. We can deal with two problems at least by solving the optimization problem, one is that we can learn the thresholds and proper cost functions from given data without any preliminary knowledge, and the other is that we can define a new attribute reduction. The attribute reduction can be interpreted as finding the minimal attribute set to make the whole decision cost minimum, which is more intuitive and reasonable.

With proper cost functions, we can derive different thresholds and get the corresponding probabilistic rough set models. The cost functions play an important role in decision-theoretic rough set model. In general, the cost functions are given by experts, but it weakens the applicability of decision-theoretic rough set model under the situation of lack of preliminary knowledge. In current researches, few contributions are on learning cost functions from data. Based on game theory, Herbert and Yao [4] proposed an approach to governing the modification of cost functions in order to improve some measures. Users need to provide some measures first and define an acceptable levels of tolerance to stop the repeating procedure. Compared to their method, our method does not need users' participation, and it is automatic and easy to implement.

As to the non-monotonic property of the regions in decision-theoretic rough set model, interpretation difficulties exist in those attribute reductions which are defined on the basis of preserving specific regions [48]. The minimum cost attribute reduction defined in this paper does not concentrate on preserving any region. Instead, the goal of the reduction is to help users make better decisions, which means less decision cost.

The rest of the paper is organized as follows. In Section 2, we review the main ideas of decision theoretic rough set model. In Section 3, we give a detailed explanation of the optimization representation, and by solving an optimization problem, we can learn the thresholds and the cost functions from data. An adaptive learning algorithm and a genetic approach are proposed. We also define a new attribute reduction and design two feasible approaches. Section 4 gives experimental results and discusses some remarks on the optimization representation. Section 5 concludes.

2. Basic notions of decision-theoretic rough set model

In this section, we present some basic definitions of decision-theoretic rough set model [41].

Definition 1. A decision table is the following tuple:

$$S = (U, At = C \cup \{D\}, \{V_a | a \in At\}, \{I_a | a \in At\}), \tag{1}$$

where U is a finite nonempty set of objects, At is a finite nonempty set of attributes, C is a set of condition attributes describing the objects, and D is a decision attribute that indicates the classes of objects. V_a is a nonempty set of values of $a \in At$, and $I_a : U \to V_a$ is an information function that maps an object in U to exactly one value in V_a.

In the decision table, an object x is described by its equivalence class under a set of attributes $A \subseteq At$: $[x]_A = \{y \in U | \forall a \in A(I_a(x) = I_a(y))\}$. Let $\pi_D = \{D_1, D_2, \ldots, D_m\}$ be a partition of the universe U defined by the decision attribute D.

Let $\Omega = \{\omega_1, \ldots, \omega_s\}$ be a finite set of s states and let $\mathcal{A} = \{a_1, \ldots, a_m\}$ be a finite set of m possible actions. Let $\lambda(a_i|\omega_j)$ denote the cost, for taking action a_i when the state is ω_j. Let $p(\omega_j|x)$ be the conditional probability of an object x being in state ω_j, suppose action a_i is taken. The expected cost associated with taking action a_i is given by:

$$\mathcal{R}(a_i|x) = \sum_{j=1}^{s} \lambda(a_i|\omega_j) \cdot p(\omega_j|x). \tag{2}$$

In decision-theoretic rough set model, the set of states $\Omega = \{X, X^c\}$, indicating that an object is in a decision class X and not in X, respectively. The probabilities for these two complement states can be denoted as $p(X|[x]) = \frac{|X \cap [x]|}{|[x]|}$