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Rule acquisition and complexity reduction in formal decision contexts

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ABSTRACT

In this paper, we introduce the notion of formal decision context as an extension of formal contexts by employing the notion of decision information table. We use formal concept analysis to formulate an approach to extract "if-then" rule from formal decision contexts. We also construct a knowledge-lossless method for complexity reduction in formal decision contexts so that the maximum rules extracted from the reduced formal decision contexts are identical to that extracted from the initial decision formal contexts. More specifically, we develop the discernibility matrix and the discernibility function in formal decision contexts to compute all of the attribute reductions without loss of knowledge.

1. Introduction

Formal concept analysis (FCA) [17,51] advances mathematical thinking for conceptual data analysis and knowledge processing. The notion of formal concept has its origin in formal logic. The adjective "formal" in formal concept means that the concept is a rigorously defined mathematical object. The main concerns in FCA are those of formal concepts and concept lattice, which can be used for the conceptual unfolding of data. FCA is a useful tool for knowledge representation and knowledge discovery, and has been widely applied to data mining, knowledge acquisition, database management systems, automatic classification, software engineering, and other disciplines in the last decade [6,7,10,15,20,24,28,33,47].

A growing number of studies within the field deals with the both theoretical and practical issues. For example, in many practical applications, several generalizations to FCA can be found in the literature [12,14,16,29]. Triadic concepts, where a concept consists of three sets of objects, attributes, and conditions under which objects may possess certain attributes were introduced in [29]. A theoretical model to extend FCA to any kind of lattice-structured set of properties was presented in [12]. A generalization replacing attributes by logic expressions was discussed in [16]. Monotone concepts were introduced in [14]. Furthermore, in more general cases, FCA was generalized to the fuzzy formal contexts [2–5,8,9,18,41], i.e., fuzzy FCA. Burusco [8,9] generalized the model of FCA based on fuzzy formal context. Based on residuated lattice, Belohlavek [2–5] proposed fuzzy concepts in fuzzy formal contexts. Georgescu and Popescu [18,41] discussed a general approach to fuzzy FCA. Yahia et al. [56] and Krajči [25] independently proposed a "one-sided fuzzy approach". Zhang et al. [62] further constructed "variable threshold concept lattices", i.e., crisp-fuzzy variable threshold concepts and fuzzy-crisp variable threshold concepts, in which the "one-sided fuzzy approach" becomes a special case. Wu and Liu [52] proposed real formal concept analysis based on grey-rough set theory.

FCA is analyzed on the basis of the notion of formal context. In fact, a formal context corresponds to an information system with the domain {0, 1}. As we know, most applications, such as classification, machine learning, decision support

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and knowledge discovery problems, can fall into the attribute-value representation model, i.e., an information system. By dividing the attributes into condition attributes and decision attributes, then such an information system is called a decision information system. Knowledge hidden in data contained in decision information systems may be discovered and expressed in the form of "if-then" rules. Because of various factors such as noise in data, compact representation, prediction capability, etc., most decision information systems are not consistent. In order to deal with uncertainty, many researchers introduced probabilistic approaches to Pawlak rough set theory, and some generalized rough set models were proposed and studied intensively. For example, the decision-theoretic rough set model (DTRSM) [35,37,57,60,61]; the variable precision rough set model (VPRSM) [65,66]; and the game-theoretic rough set model (GTRSM) [21,22]. As a meaningful research direction, recently, DTRSM has received wide attention and is studied intensively. For example, Yao [60] presented a detailed analysis of three-way decision rules in the DTRSM and applied three-way decision rules for two-category classification. In the literature [61], Yao further compared probabilistic three-way decisions, probabilistic two-way decisions, and qualitative three-way decisions of the standard rough set model, and showed that the three-way model is superior to other two models under certain conditions. Qian et al. [44] developed a frame work of multigranulation decision-theoretic rough sets, in which lower/upper approximations are approximated by granular structures induced by multiple binary relations. Jia et al. [23] proposed an optimization representation of decision-theoretic rough set model. In the literature [34], the triangular fuzzy decision-theoretic rough sets model is proposed by considering the loss functions being expressed by triangular fuzzy numbers. Azam and Yao [1] used the GTRSM to construct a mechanism for analyzing the uncertainties of rough set regions in the aim of finding effective threshold values. Li and Yang [36] proposed an axiomatic approaches to probabilistic rough sets. In the literature [19], Grzymala-Busse et al. discussed a generalization of probabilistic approximations of incomplete data, and compared experimentally two generalizations of probabilistic approximations: global and local.

Knowledge acquisition and knowledge reduction are central issues of information system, and are outstanding contribution made by Rough Set [40] research to data analysis. Based on different binary relation and different requirements, many types of attribute reduction have been proposed in the analysis of information systems and decision tables [26,30–32,42,43,48,53,55,58,59,64]. In terms of formal contexts, knowledge reduction is the search for a minimal attribute subset that preserves required properties, and non-essential attributes can be deleted from the databases. After knowledge reduction, knowledge hidden in formal contexts can be discovered and expressed in the form of decision rules. In [17], Gediga and Wille discussed the attribute reduction of formal contexts from the viewpoint of reducing columns and rows, but without a concrete reduction approach and algorithm. In [63], Zhang and Wei discussed attribute reduction of classical formal contexts, and formulated the reduction approach and the Boolean algorithm. Wang and Zhang discussed the relations of attribute reduction between object and property oriented concept lattices by keeping meet-irreducible elements [49]. Liu et al. [38] presented a reduction method for concept lattices based on rough set theory and its application. Cheung and Vogel [11] used the idea of quotient lattice to reduce the complexity of a Term-Document Lattice. Based on fuzzy K-means clustering, Kumar and Srinivas [27] proposed a method to reduce the size of the concept lattices by employing corresponding object-attribute matrix. Wu et al. [54] studied knowledge reduction in formal contexts from the viewpoint of preserving granular structure of concept lattices. Wei et al. [50] showed an attribute approach of concept lattice based on formal decision contexts. Liu et al. [39] showed an efficient post-processing method to prune redundant rules in virtue of the property of Galois connection, which inherently constrains rules with respect to objects. In the paper, the methods to rule acquisition and knowledge reduction in formal contexts are investigated by employing rough set approaches of rule acquisition and knowledge reduction.

Reduction approaches in both Wu et al. [54] and Wei et al. [50] require formal decision contexts to be consistent, i.e., satisfying a partial order between the conditional concept lattice and decision concept lattice. They are, however, and does not suitable for inconsistent formal decision contexts. However, in most situations, formal decision contexts are inconsistent and this limits its applications. At present, knowledge reduction for inconsistent formal decision contexts has not been examined. The main purpose of this paper is to propose a more general approach to rule acquisition and knowledge reduction in formal decision contexts (consistent and inconsistent). By using the theory and method of decision information system, in this paper, we propose methods to extract “if-then" rule from the decision formal contexts. Inspired by the analysis of Cheung and Vogel [11], we also formulate a knowledge-lossless approach to complexity reduction in formal decision contexts, by which the complexity of concept lattice is substantially reduced. The proposed reduction approach is also suitable for inconsistent formal decision contexts.

The structure of the paper is as follows: In Section 2, we introduce the notion of formal decision context and ways to extract “if-then" rules from the formal decision contexts. Section 3 discusses our approach to complexity reduction in the formal decision contexts. In Section 4, we construct the discernibility matrices and discernibility functions for the formal decision contexts. We then conclude the paper with a summary in Section 5.

2. Rules in formal decision contexts

A formal context is a triplet $\mathbb{K} = (U, A, I)$, where U is a non-empty finite set of objects and A is a non-empty finite set of attributes, I is a relation between U and A, which is a subset of the Cartesian product $U \times A$. For a pair of elements $x \in U$ and $a \in A$, if $(x, a) \in I$, we say “object x has attribute a", or “attribute a is possessed by object x".