



Approximation and robustness of fuzzy finite automata [☆]

Yongming Li ^{*}

College of Computer Science, Shaanxi Normal University, Chang'an South Street No. 199, Xi'an, Shaanxi 710062, China

Received 30 July 2006; received in revised form 6 May 2007; accepted 24 May 2007

Available online 2 June 2007

Abstract

In previous work we have shown that *nondeterministic fuzzy finite automata* (or NFFAs, for short) under max-* compositional inference for some *t*-norm * and *deterministic fuzzy finite automata* (or DFFAs, for short) are not necessarily equivalent. We continue to study the approximation and robustness of fuzzy finite automata in this paper. In particular, we show that we can approximate an NFFA by some DFFA with any given accuracy when the NFFA is not equivalent to any DFFA, and the related construction is also presented. Some characterizations of NFFA and DFFA are given. We study the robustness of fuzzy finite automata against imprecision of fuzzy transitions, and some interesting results are obtained.

© 2007 Elsevier Inc. All rights reserved.

Keywords: Nondeterministic fuzzy finite automaton; Deterministic fuzzy finite automaton; Approximation; Robustness

1. Introduction

We can use different fuzzy inference methods in constructing a fuzzy system [24,15]. The most commonly used fuzzy inference method is known as the compositional rule of inference (or CRI) and its generalization max-* compositional inference for some *t*-norm *. We can use different kinds of *t*-norms when we implement max-* compositional inference in the practical processes. However, the resulting systems are equivalent in the approximate sense. That is to say, for any practical process, if we can approximately describe the process by a fuzzy system with max-min compositional inference, we can always approximately describe it by a fuzzy system with max-* compositional inference for any other *t*-norm *.

Fuzzy finite automata can be seen as a kind of discrete fuzzy system [20,15,6] that can describe practical processes. In particular, there are a large number of discrete event systems to be treated in a manner of computing with words when describing their states, this is especially true in biomedical applications, as stated in

[☆] This work is supported by National Science Foundation of China (Grant No. 10571112), National 973 Foundation Research Program (Grant No. 2002CB312200) and Key Research Project of Ministry of Education of China (No. 107106).

^{*} Tel.: +86 29 85307628; fax: +86 29 85310161.

E-mail address: liyongm@snnu.edu.cn

[19]. A typical example is a person's health status, where the change of the condition of a person's health from a state, say "excellent", to another, say "good", is obviously imprecise, since it is hard to measure exactly the change. In reality, this representation leads to the formulation of fuzzy finite automata and fuzzy discrete event systems [19]. Adamy and Kempf [2] introduced a method to model discrete-time recurrent fuzzy systems by fuzzy finite automata, which yields both a clear picture of the dynamics of recurrent fuzzy systems and a methodology for designing them using fuzzy automata. In practice, fuzzy automata and fuzzy languages have been used to solve meaningful problems such as intelligent interface design [10], clinical monitoring [22], neural networks [7], pattern recognition [20], chemical reactions, mobile robots in an unstructured environment [3], intelligent vehicle control [21], waste-water treatment [23], and web intelligence [25] in which the state transitions of some systems are always somewhat imprecise, uncertain, and vague.

In the modeling these practical processes, we can use different fuzzy finite automata. The most commonly used fuzzy model is known as the fuzzy automaton with max-* composition for some t -norm *. Theoretically, for every t -norm *, there will be a corresponding type of fuzzy automaton using max-* compositional inference that could be used in modeling practical processes. For example, we can use fuzzy automata to model discrete event systems with fuzzy uncertainty, as done in [19] using a *nondeterministic fuzzy finite automaton* (or NFFA, for short) with max-product compositional inference, while [5] using an NFFA with max-min compositional inference. Adamy and Kempf [2] used a fuzzy finite automaton with max-product composition to model discrete-time recurrent fuzzy systems.

A crucial theoretical problem is whether fuzzy finite automata using max-* compositional inference are equivalent for modeling practical processes. Contrary to ordinary case, we have shown that a nondeterministic fuzzy finite automaton (NFFA) under max-* compositional inference is in general not equivalent to a *deterministic fuzzy finite automaton* (or DFFA, for short) [13] in processing fuzzy languages. Some necessary and sufficient conditions for the equivalence between NFFA and DFFA are given in [13,17,18]. So it is necessary to require the proposed models of NFFAs using max-* compositional inference for different t -norms * to be equivalent in the approximate sense. We express this problem explicitly as follows: for any discrete practical process, if we can approximately describe the process by a fuzzy finite automaton with max-min compositional inference, can we always approximately model it by a fuzzy finite automaton with max-* compositional inference for any other t -norm *? We have proved that DFFA and NFFA under max-min compositional inference are equivalent. In order to discuss whether NFFAs using max-* compositional inference for different t -norms are equivalent in the approximate sense, we only need to study the approximation property of NFFA by DFFA. This forms the first topic of this study.

Another issue of the paper is to consider the robustness of fuzzy finite automata against imprecision of fuzzy transitions. Intuitively, this is an important issue for fuzzy systems and fuzzy reasoning. Since approximation errors appear in the formulation of fuzzy models of real systems, it is necessary to require the models of fuzzy systems to be tolerant of approximation errors. As a model of real systems, fuzzy finite automata should be robust for some minor approximation errors, which forms another topic of this paper.

The content of this paper is arranged as follows. In Section 2, we first recall the definition of NFFA and DFFA. The relationship between NFFA under max-min compositional inference and DFFA is also discussed. We show that NFFA under max-* compositional inference can be approximated by DFFA with any given accuracy for any t -norm * satisfying the weakly finite generated condition (in short, WFGC), even if they are not equivalent in general. In Section 3, we study the robustness of NFFAs. Some conclusions are presented finally.

2. Approximation of nondeterministic fuzzy finite automata by deterministic fuzzy finite automata

A *fuzzy finite automaton* (FFA) is seen as a *nondeterministic finite automaton* (NFA) with a fuzzy set of state transitions, i.e., to each state transition its so-called membership (or truth) degree – a number between 0 and 1 – is assigned. Following each computational path a membership degree of reaching a certain accepting state can be computed from the truth degrees of individual transitions leading to that accepting state. The formula for composing the resulting value of the membership degree of a computation depends on t -norm employed in the composition.

Download English Version:

<https://daneshyari.com/en/article/397555>

Download Persian Version:

<https://daneshyari.com/article/397555>

[Daneshyari.com](https://daneshyari.com)