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A belief revision framework for revising epistemic states with partial epistemic states ☆

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ABSTRACT

Belief revision performs belief change on an agent's beliefs when new evidence (either of the form of a propositional formula or of the form of a total pre-order on a set of interpretations) is received. Jeffrey's rule is commonly used for revising probabilistic epistemic states when new information is probabilistically uncertain. In this paper, we propose a general epistemic revision framework where new evidence is of the form of a partial epistemic state. Our framework extends Jeffrey's rule with uncertain inputs and covers well-known existing frameworks such as ordinal conditional function (OCF) or possibility theory. We then define a set of postulates that such revision operators shall satisfy and establish representation theorems to characterize those postulates. We show that these postulates reveal common characteristics of various existing revision strategies and are satisfied by OCF conditionalization, Jeffrey's rule of conditioning and possibility conditionalization. Furthermore, when reducing to the belief revision situation, our postulates can induce Darwiche and Pearl's postulates C1 and C2.

1. Introduction

Information used in real applications is often uncertain, which reflects a kind of reliability of sources and sensors. In addition, knowledge bases are not static. There is always new information that should be taken into account. In probabilistic frameworks, these two aspects (i.e., uncertain and dynamic) are handled in homogeneous way by representing uncertainty associated with information in the form of probability distributions, and using different forms of conditioning for updating.

In Artificial Intelligent (AI) community, since 1985, the process of changing beliefs with new information is known as belief revision. Belief revision [1,28,34] performs belief change on an agent's beliefs when new evidence is received. It has been observed that a pure logic-based revision framework, e.g., AGM postulates based framework, may permit some counterintuitive results in iterated revision.[1] As a result, revision on epistemic states should be introduced accordingly [14,4,51,5,33,42], etc.

☆ This paper is an extended version of a conference paper [44].

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[1] Note that AGM postulates are not designed for iterated revision. So it is not surprising they permit counterintuitive results in iterated revision.

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However, in most of these research efforts, new evidence is still represented as a *propositional* formula, not an epistemic state (even if initial epistemic state may be a propositional formula or a totally pre-ordered relation on a set of possible worlds). Therefore, these methods do not fully implement a revision that reflects the effect of epistemic states, e.g., new information could be uncertain [14,15]. Although an effort has been made to address this problem in a couple of papers (e.g., [4]), in which new evidence is represented as a full epistemic state. The revision methods proposed still cannot manage the strengths over partitions on a set of interpretations, which, in probability or possibility settings, is already accomplished by Jeffrey's rule [3]. That is, we need to develop a revision framework which can deal with new information with strengths that could be modeled by partial epistemic states similar to the probability counterparts of Jeffrey's rule. Here we should note that new information not only comes from observations of the agent on the environment but can also be conveyed by other agents where *epistemic* makes sense. So here we use partial epistemic states as inputs which we aim to cover both situations, even if there might be some abuse of concept.

Jeffrey's rule is widely applied when an agent's current belief and new evidence are both modeled in probability measures. More precisely, in Jeffrey's rule, the prior state is a probability distribution representing an agent's current beliefs or generic knowledge whilst new evidence is a partial probability measure solely on a partitioned subsets of the world. Similar strategies were also proposed for ordinal conditional functions (OCFs) [58,59], for possibility measures [19,3], etc. However, despite of the need to handle new, input information with strengths that may be present in different forms, to the best of our knowledge, there does not exist a common revision strategy (and its corresponding postulates) to address this issue. In another words, can we develop a general revision framework that subsumes these individual revision strategies (in different frameworks) with a set of common postulates? A significant advantage of this, if achievable, is to facilitate further understanding of the nature of revision, regardless of which formalism may be deployed to represent an agent's beliefs and new uncertain evidence.

To answer this question, we first propose a framework to represent an agent's epistemic beliefs, which generalizes various definitions of epistemic states in the literature (e.g., a weighted formula [33,48], a total pre-order [4], an OCF-based epistemic state [49,58,59], a probability measure [30], a partial pre-order [45,41], a mass function [46,47], etc.). This framework takes inspirations from Jeffrey's rule of conditioning under uncertain inputs. We then investigate how a set of rational postulates should be derived to regulate revision operators defined from this framework and provide representation theorems for these postulates. We prove that these postulates are satisfied by OCF conditionalization, possibility conditionalization, and most significantly Jeffrey's rule of conditioning.

Our main objective of defining a general iterated revision framework is to implement the revision of an agent's current beliefs (represented as a full epistemic state) with new, uncertain evidence (represented as a partial epistemic state). In standard AGM framework, there are no explicit representations of strengths associated with initial beliefs and inputs (even if any revision operator that satisfies AGM postulates is implicitly based on some total pre-order on interpretations). Our framework, however, supports the explicit representation of strengths which will help in determining the result of revision.

Furthermore, we investigate the relationships between this general framework with logic-based belief/epistemic revision, especially with Darwiche and Pearl's (DP's) belief revision framework [14]. We prove that when reducing to the belief revision situation, our postulates can induce DP's postulates C1 and C2.

To summarize, the main contributions of the paper are:

- Our definition of epistemic states subsumes many existing definitions of epistemic states.
- We provide a generalized revision strategy and corresponding postulates.
- We prove two presentations theorems which show clear and succinct *kinematic* semantics in revision.
- We prove that our framework can recover many existing numerical revision operators, e.g., Jeffrey's rule, OCF conditionalization, possibilistic revision, etc.
- When an input is a formula, our postulates can induce all the AGM–KM postulates and Darwiche and Pearl's C1, C2 postulates.

In other words, our framework provides an important one step forward of extending the various existing revision strategies and revision operators.

The rest of the paper is organized as follows. We provide the preliminaries and Jeffrey's rule in Sections 2 and 3 respectively. In Section 4, formal definitions of epistemic space and epistemic state are introduced. In Section 5, we propose a set of postulates for epistemic revision and their corresponding representation theorems. In Section 6 and Section 7, we discuss how our framework subsumes existing revision strategies. Finally, we conclude the paper in Section 8.

2. Preliminaries

We consider a propositional language $\mathcal{L}$ defined on a finite set $\mathcal{A}$ of propositional atoms, which are denoted by p, q, r, etc. A proposition ϕ is constructed by propositional atoms with logic connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$ in the standard way. An interpretation ω (or possible world) is a function that maps $\mathcal{A}$ onto the set $\{0, 1\}$. The set of all possible interpretations on $\mathcal{A}$ is denoted as W. Function ω can be extended to any proposition in $\mathcal{L}$ in the usual way. An interpretation ω is a model of (or satisfies) ϕ iff $\omega(\phi) = 1$, denoted as $\omega \models \phi$. We use $Mod(\phi)$ to denote the set of models for ϕ. We write $\phi \vdash \psi$ if $Mod(\phi) \subseteq Mod(\psi)$ and $\phi \equiv \psi$ if $Mod(\phi) = Mod(\psi)$. Furthermore, we also take the view that a proposition ϕ can