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Simulation for lattice-valued doubly labeled transition systems

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ABSTRACT

During the last decades, a large amount of multi-valued transition systems, whose transitions or states are labeled with specific weights, have been proposed to analyze quantitative behaviors of reactive systems. To set up a unified framework to model and analyze systems with quantitative information, in this paper, we present an extension of doubly labeled transition systems in the framework of residuated lattices, which we will refer to as lattice-valued doubly labeled transition systems (LDLTSs). Our model can be specialized to fuzzy automata over complete residuated lattices, fuzzy transition systems, and multi-valued Kripke structures. In contrast to the traditional yes/no approach to similarity, we then introduce lattice-valued similarity between LDLTSs to measure the degree of closeness of two systems, which is a value from a residuated lattice. Further, we explore the properties of robustness and compositionality of the lattice-valued similarity. Finally, we extend the Hennessy–Milner logic to the residuate lattice-valued setting and show that the obtained logic is adequate and expressive with lattice-valued similarity.

1. Introduction

Many real-life systems, such as multimedia equipments, communication protocols and fault-tolerant systems, are reactive systems [1,3,13,28] that compute by reacting to stimuli from their environment. Various transition systems, such as, automata, labeled transition systems (LTSs), Kripke structures (KSs), and more generally, doubly labeled transition systems (DLTSs) [29] also known as labeled Kripke structures [9], have been successfully applied in formal verification of reactive systems. It is well known that simulation [28] and temporal logics [3,13] are two principal verification methods that are used for analyzing transition systems.

Initially, transition systems were mainly designed to model and assess qualitative information of reactive systems. In the literature up to now, a great variety of transition systems whose transitions or states carry weights in a multi-valued setting have been proposed and many of these have been actually used for verification purposes. Automata taking membership values in a complete residuated lattice were first studied by Qiu in [33] and further studied in [34,38]. From a different point of view, Ćirić and his colleagues [11,12] introduced two types of simulation and four types of bisimulations for fuzzy automata over a complete residuated lattice. Furthermore, some related algorithms for computing the greatest simulations and bisimulations have been well developed. More recently, the Cao et al. investigated bisimulation for fuzzy transition systems [7]. Model checking problems for multi-valued versions of the classical logics LTL, CTL, and μ-calculus, which are

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interpreted over multi-valued Kripke structures taking values in a finite De Morgan algebra, have been extensively studied (see, for example, [10,23,36]). Moreover, Kupferman and Lusting [24] defined a lattice-valued simulation between multi-valued Kripke structures. They showed that the lattice-valued simulation can be logically characterized by the universal fragment of lattice-valued μ-calculus and calculated in polynomial time.

The purpose of this paper is to set up a unified extension of transition systems and to provide a uniform definition of simulations. We defer the treatments of bisimulation and language theories to other work. A unified framework is helpful to unify system analysis. The main challenge in doing so is to make a comprise between the generality of models as well as simulations and the preservation of lattice-valued analogues to boolean properties of simulations.

We present an extension of doubly labeled transition systems, called lattice-valued doubly labeled transition systems (LDLTSs). Contrary to the classic DLTSs, the main features of LDLTSs are that LDLTSs decorate each transition with a weight in a residuated lattice, an atomic proposition at every state is interpreted as an element of the residuated lattice, and the set of actions is equipped with a lattice-valued equality relation. Our model can be specialized to fuzzy automata over complete residuated lattices, fuzzy transition systems, and multi-valued Kripke structures. For example, if the set of actions of an LDLTS is a singleton, then it is just a multi-valued Kripke structure. The reasons for considering our model based on DLTS are as follows: (1) KSs and LTSs are both the special cases of DLTSs. Thus, being once proved in LDLTSs, the results will be valid in the lattice-valued extensions of KSs and LTSs. (2) DLTSs have some important applications in several different areas. For example, DLTS is a semantics domain of state-based and action-based temporal logics, such as, state/event linear temporal logic [9], branching time temporal logics SocL [16] and UCTL [4]. Thus, theory obtained over the LDLTSs may have some important applications in different areas.

We will use residuated lattices as the structures of truth values of LDLTSs. One reason comes from the fact that residuated lattices are general algebraic structures and generalize many algebras with very important applications [17,22,30,31,35]. Some well-known algebraic structures, such as Heyting algebras [5], BL algebras [19] and MTL algebras [15], are all particular cases of residuated lattices. Another reason is that the richness of the structure of residuated lattices allows us to interpret operators in temporal logics and establish the quantitative verification relations between a system and its specification.

To be able to reason not only whether a given implementation satisfies the specification, but also to what extent, we introduce lattice-valued similarity between LDLTSs that measures the degree of closeness of two systems as a value taken from a residuated lattice, in contrast to the traditional yes/no approach to simulation. The notion of our similarity follows the methodology described in [24]. Loosely speaking, lattice-valued similarity in [24] and the greatest forward simulation in [11,12] are both special cases of our similarity.

We then show that our similarity has some appealing properties that are necessary for quantitative verification: (1) Our similarity is a lattice-valued preorder relation, hence our notion can be used to as a method for implementation verification; (2) In application the proposition values, lattice-valued equality relation over the set of actions, together with lattice-valued transition relation, of an LDLTS are somewhat imprecise and subjective, because they are often provided by the experts in ad hoc (heuristic) manner from experience or intuition. Hence, we discuss the robustness of our notions based on a logically equivalence measure [5,14]; (3) For a synchronous composition operator, we show that our similarity is compositional. However, these properties have not been studied in [24] and cannot be obtained, because the structure of the truth values of LDLTSs—residuated lattice ensures the existence of these properties, but, finite De Morgan algebra defined as in [24] cannot constitute a residuated lattice.

We also consider a logical characterization of our similarity. The logical characterization of lattice-valued similarity consists of two aspects: adequacy and expressivity, whereas expressivity of logical characterization of lattice-valued similarity of [24] has not been mentioned. In [11,12] the authors have left out logical characterizations of their (bi)simulations unexplored. Recall that in the two-valued setting [3], a logic A is adequate when one state s_1 of Kripke structure is simulated by another state s_2 if and only if the properties expressed by the A formulas that are satisfied in s_2 are also satisfied in s_1. The logic A is expressive when each state s_2 has a characteristic formula φ_{s_2} in A such that s_2 simulates s_1 if and only if s_1 satisfies φ_{s_2}. Naturally, we want to have the corresponding results in our framework. Through extending the Hennessy–Milner logic HML [20] to the lattice-valued setting, we obtain the lattice-valued Hennessy–Milner logic LHML and show two fragments of LHML are adequate and expressive with our similarity relation. These results show that lattice-valued similarity can be used to quantitative refinement and abstraction in the context of multi-valued model checking.

The paper is structured as follows. We review some basic facts on residuated lattices and introduce the notion of LDLTSs in Section 2. The notion of lattice-valued similarity over LDLTSs is defined and its properties are discussed in Section 3. We relate the lattice-valued similarity with lattice-valued Hennessy–Milner logic in Section 4, and conclude the paper in Section 5.

2. Preliminaries

In this section, after briefly recalling a few basic facts on residuated lattices, we present lattice-valued doubly labeled transition systems LDLTSs as a quantitative model for reactive systems. We write $\mathbb{N}$ for the set of natural numbers and I the index set.

A *residuated lattice* [5,19] is an algebra $\mathcal{L} = (L, \wedge, \vee, \otimes, \rightarrow, 0, 1)$, where $(L, \wedge, \vee, 0, 1)$ is a bounded lattice with the least element 0 and the greatest element 1; $(L, \otimes, 1)$ is a commutative monoid, and $\otimes$ and $\rightarrow$ satisfy the adjointness property, i.e. $x \otimes y \leqslant z$ if and only if $x \leqslant y \rightarrow z$, where $x, y, z \in L$. The binary operations $\otimes$ and $\rightarrow$ are called product and implication,