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Set-based granular computing: A lattice model

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ABSTRACT

Set-based granular computing plays an important role in human reasoning and problem solving. Its three key issues constitute information granulation, information granularity and granular operation. To address these issues, several methods have been developed in the literature, but no unified framework has been formulated for them, which could be inefficient to some extent. To facilitate further research on the topic, through consistently representing granular structures induced by information granulation, we introduce a concept of knowledge distance to differentiate any two granular structures. Based on the knowledge distance, we propose a unified framework for set-based granular computing, which is named a lattice model. Its application leads to desired answers to two key questions: (1) what is the essence of information granularity, and (2) how to perform granular operation. Through using the knowledge distance, a new axiomatic definition to information granularity, called generalized information granularity is developed and its corresponding lattice model is established, which reveal the essence of information granularity in set-based granular computing. Moreover, four operators are defined on granular structures, under which the algebraic structure of granular structures forms a complementary lattice. These operators can effectively accomplish composition, decomposition and transformation of granular structures. These results show that the knowledge distance and the lattice model are powerful mechanisms for studying set-based granular computing.

1. Introduction

Granular computing (GrC) originally proposed by Zadeh [54] plays a fundamental role in human reasoning and problem solving (see [48]). Its three basic issues are information granulation, organization and causation. As pointed out in [53–57], the information granulation involves decomposition of whole into parts; the organization involves integration of parts into whole; and the causation involves association of causes with effects. They have been applied in relevant fields such as interval analysis, cluster analysis, machine learning, databases, data mining and knowledge discovery. The research on granular computing has led to four important methods, which are rough set theory [8,10–12,25,26,42,60], fuzzy set theory [13,14,24], concept lattice theory [22,23,40,45], and quotient space theory [58].

Set-based granular computing is a type of granular computing whose concerned universe is characterized by a finite set. It is a dominant research task in granular computing. The following concepts are elementary for set-based granular

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computing. *A granule* is a clump of objects or points drawn together by indistinguishability, similarity, and proximity of functionality [57]. Granulation of the universe leads to a collection of granules. *A granular structure* is a mathematical structure of the collection of granules, in which the inner structure of each granule is visible (a granule is a white box) and the interactions among granules are detected by the visible structures. *A knowledge base* is a collection of some granular structures on the same universe. Given these concepts, an important task is to establish a conceptual framework for granular computing. For our further development, we will briefly review several focuses of attention in granular computing. They include a measure of granularity [15–17,31,36,43,46], information processing [21], a framework of granular computing [18,20,33], problem solving based on "granulate and conquer" principle and quotient theory [60], dynamic granulation [32,35], multigranulation view [34,37], granular computing for human-centric information processing [1–3,52], and their applications [27–29,59]. Yao et al. [49] have reviewed perspectives and challenges in granular computing in the first ten years of its development. It can be seen from the developments that granular computing has been evolving into a field of cross-disciplinary study. In set-based granular computing, there are three key issues to address, which are information granulation, information granularity and granular operation.

Information granulation is to construct a set of information granules by a given granulation strategy in set-based granular computing. It is a pretreatment step of granular computing and the basis of problem solving with it. Information granulation will adopt various strategies according to user's acquirements and targets of problem solving. Common granulation strategies are mainly based on binary relations [8,10–12,16,21], clustering analysis [4], proximity of functionality [6] and image segmentation [7], and so on. In a broad sense, each of granulation strategies can be seen as the information granulation induced by a specific binary relation. For example, image segmentation is to partition an image to a set of sub-images in which each sub-image can be seen as an equivalence class, and hence this strategy can be characterized by an equivalent relation. The granulation of objects induced by an equivalent relation is a set of equivalence classes, in which each equivalence class can be regarded as a (Pawlak) information granule [25,26]; the granulation of objects induced by a tolerance relation generates a set of tolerance classes, in which each tolerance class also can be seen as a tolerance information granule [16]. By using a neighborhood relation, objects are granulated into a set of information granules, and each neighborhood is called a neighborhood information granule [19,44]. Dick et al. [9] proposed a granular neural network, and researched its performance analysis and application to re-granulation. However, these granular structures have no unified knowledge representation, which could be inefficient to some extent for establishing a conceptual framework for set-based granular computing.

Information granularity in set-based granular computing is an index to measure the granulation degree of objects in a given data set. How to calculate the information granularity of a granular structure has always been a key problem. In general, the information granularity represents discernibility ability of information in a granular structure. The smaller the information granularity, the stronger its discernibility ability. To date, several forms of information granularity have been proposed according to various views and targets [15,17,31,43,46]. Wierman [43] introduced the concept of granulation measure to quantify the uncertainty of information in a knowledge base. This concept has the same form as Shannon's entropy under the axiom definition. Liang et al. [15,16] proposed information granularity in either of complete and incomplete data sets, which have been effectively applied in attribute significance measure, feature selection, rule extraction, etc. Qian and Liang [31] presented combination granulation with an intuitive knowledge-content nature to measure the size of information granulation in a knowledge base. Xu et al. [46] gave an improved measure for roughness of a rough set in rough set theory proposed by Pawlak [25], which is also an information granularity in a broad sense. In the above forms of information granularity, the partial order relation plays a key role in characterizing the monotonicity of each of them. Although these excellent research contributions have been made in the context of set-based granular computing, there remains an important issue to be addressed. What is the essence of measuring an information granularity? As mentioned by Zadeh, in general, information granularity should characterize the granulation degree of objects from the viewpoint of hierarchy [54]. This provides a point of view that an information granularity should characterize hierarchical relationships among granular structures. To answer the question, in this investigation, we will develop an axiomatic approach to information granularity in set-based granular computing.

Granular operation is to answer the problem how to achieve composition, decomposition and transformation of information granules/granular structures. This problem is also one of key tasks in set-based granular computing [18–21,33,44,47,50,51]. Granular operation has two types of operations. One is operation of information granules, and the other is operation of granular structures. Lin [18–21] proposed a kind of operators among information granules, called knowledge operation, in which each information granule is a neighborhood. Yao [50,51] gave another approach to operation of information granules in a neighborhood system. Wu et al. [44] investigate how to perform operation of information granules in k-step-neighborhood systems. Yang et al. [47] modified Lin's version for much better operation of information granules. If we see operation of information granules in the context of granular structures, these operators can be used to generate new granular structures, and hence operation of information granules can be seen as inner operation of granular structures [33]. To perform operation of granular structures, Qian et al. [33] proposed four operators on tolerance granular structures. This is an attempt to study composition, decomposition and transformation of granular structures in set-based granular computing. In this study, we focus on the second granular operation, i.e., operation of granular structures with the unified knowledge representation.

From the research progresses above, it can be seen that many important results have been developed in the literature, however, there is no unified framework for these developments, which could be inefficient to some extent for studying