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Evidence-theory-based numerical algorithms of attribute reduction with neighborhood-covering rough sets

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ABSTRACT

Covering rough sets generalize traditional rough sets by considering coverings of the universe instead of partitions, and neighborhood-covering rough sets have been demonstrated to be a reasonable selection for attribute reduction with covering rough sets. In this paper, numerical algorithms of attribute reduction with neighborhood-covering rough sets are developed by using evidence theory. We firstly employ belief and plausibility functions to measure lower and upper approximations in neighborhood-covering rough sets, and then, the attribute reductions of covering information systems and decision systems are characterized by these respective functions. The concepts of the significance and the relative significance of coverings are also developed to design algorithms for finding reducts. Based on these discussions, connections between neighborhood-covering rough sets and evidence theory are set up to establish a basic framework of numerical characterizations of attribute reduction with these sets.

1. Introduction

The theory of rough sets [15] is a well-known mathematical approach to address inexact, uncertain or insufficient data and has successfully been applied to many practical problems in machine learning, pattern recognition, decision analysis, process control, knowledge discovery in databases, and expert systems. In fact, partitions and equivalence relations are the principle concepts in classical rough set theory. The definitions of the lower and upper approximations of an arbitrary set are given based on an equivalence relation or partition, and the attribute reduction of datasets are developed in terms of approximations in rough sets to delete the superfluous attributes in a dataset. The concept of attribute reduction can be viewed as the strongest and most important result in rough set theory, and this result distinguishes this theory from other theories.

Classical rough sets were developed to address those datasets where each object can only take a unique discrete value for every attribute, and furthermore, every attribute in such datasets can induce a partition of the universe of discourses. However, the above conditions cannot always be satisfied in many datasets collected from practical problems. For example, it is possible that an object can take set values and have missing or real values for some attributes. Thus, datasets in which objects take different types of values rather than unique discrete values for some attributes are always available. It was reported in [6] that attributes in such datasets can often induce a covering by certain techniques rather than a partition. Moreover, Couso and Dubois [7] noted that the use of multiple-valued mapping to address incomplete information can also induce a covering. In this paper, incomplete information means a dataset in which objects can take set values for some

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attributes, and this approach is just one method to induce coverings. In Section 2, we will demonstrate how to induce coverings by set-valued attributes, missing-valued attributes and real-valued attributes. Thus, the attributes in such datasets are closely related to coverings. Furthermore, to address the attributes in these datasets by employing rough sets, covering rough sets were considered as a generalization of Pawlak rough sets in which partitions are replaced by coverings [1–7, 9–14,16–18,20,23–30,32–42]. Therefore, the research of covering rough sets is of practical significance.

Within the authors' knowledge, Zakowski [34] first proposed this generalization by employing coverings of the universe and establishing generalized rough sets. Pomykala [16] proposed two natural definitions of upper and lower approximations of a set of objects: one definition can extend the lower approximation of a set as the union of covering elements contained in the set, and the upper approximation is considered as its dual via complementation; the other definition extends the upper approximation of a set as the union of the covering elements that intersect the set, and the lower approximation is considered as its dual via complementation. Many other definitions of covering-based rough sets have been proposed. Samanta and Chakraborty [18] collected and categorized these approximations in terms of classical set theoretic properties. In addition, Couso and Dubois [7] aimed to bridge the gap between generalized rough sets based on coverings and the possible-world approach to incomplete information. Each set in a covering is the upper inverse image of an attribute's value through a multiple-valued mapping that describes incomplete knowledge of the attributes' values, and if the upper and lower approximations of a set are poorly known, this interpretive setting can be applied to choose the most appropriate covering-rough-set definitions from the different covering rough sets. These authors also concluded that the covering approximations in [16] are loose and tight pairs. Whereas Bonikowski et al. [1] studied the structures of coverings. Mordeson [13] examined the relationship between the approximations of sets defined with respect to coverings and some axioms satisfied by traditional rough sets. Moreover, Chen et al. [5] discussed covering rough set under the framework of a complete, completely distributive lattice, and Zhu [36,37] and Zhu and Wang [35,38] compared three types of generalized rough sets to address vagueness and granularity in information systems. A key concept known as "reduct" is proposed to remove the redundant members in a cover and to obtain the smallest covering that can be used to induce the same covering's lower and upper approximations in the first and third types of covering generalized rough sets. Another key concept is known as "exclusion," and it is presented to remove all the immured elements in a covering to obtain the smallest covering that can be used to induce the same lower and upper approximations in the second type of covering generalized rough sets. Furthermore, Zhang and Luo [39], Zhang et al. [40] and Yun et al. [33] considered the axiomatic approaches of covering rough sets. In [30–32], Yao summarized the existing research on the approximation operators of covering rough sets and proposed a uniformed framework for covering rough sets by elements, granules and subsystems based on the definitions of approximation operators for covering rough sets. The results in [30–32] set up a framework for covering rough sets with constructive approaches.

However, all these works were developed to discuss the properties of coverings and set approximations; they were not concerned with the attribute reduction of covering rough sets. As previously mentioned, attributes in datasets whose objects can take different types of values, and such as set values, missing values and real values, can always induces coverings. Thus, for these types of datasets, coverings should be employed to substitute for attributes when covering rough sets are employed as a mathematical tool to reduce superfluous attributes. One of the key points of attribute reduction with covering rough sets is how to aggregate several coverings into a reasonable covering. In [6], the concept of an induced covering was proposed: for a fixed covering, the elements of the induced covering are intersections of the sets in the covering that contain particular objects, and this intersection can be interpreted as the minimal description of the particular sample. Thus, the collection of observed attributes can clearly be identified with the collection of associated induced covers, and using induced coverings, we can define the intersection of induced coverings. As a result, in a natural way both notions derive the covering associated to the collection of all attributes, which is the coarsest covering that refines all the initial coverings associated to each attribute; this covering is formed by the intersections of the elements in all the initial coverings. This idea is similar to the relationship between attributes and partitions in classical rough sets. Therefore, the idea of attribute reduction is clearly related to the idea of comparisons between this finer covering and those other, coarser coverings, and this relation both sets up a criterion to delete attributes and establishes a foundation to develop attribute reduction with covering rough sets. However, there are several types of covering rough sets (as previously reviewed), and each of them can be selected to develop attribute reduction from a theoretical viewpoint. Thus, another key point of attribute reduction with covering rough sets is to select a suitable model from them. For a given covering, the neighborhood of an element is the intersection of all the sets in the covering that contain the element, and the neighborhood of all the elements of the universe forms a new covering, which is called a neighborhood covering. The so-called neighborhood-covering rough sets are in fact the covering rough sets that are based on the neighborhood covering. In [6] and this paper, neighborhood-covering rough sets [24,35] are selected to perform attribute reduction for the following reasons. Firstly, the definition of an induced covering coincides with the definition of a neighborhood covering in covering rough sets. Furthermore, in [6] we studied attribute reduction with covering rough sets using induced coverings. Thus, it is natural to employ neighborhood-covering rough sets to develop attribute reduction with covering rough sets. Secondly, the neighborhood-covering rough sets belong to the framework of granule-based covering rough sets [30] with clear granular structures, i.e., every sample is assigned a neighborhood as its minimal description. This assignment offers mathematical techniques to benefit the computing of reducts by using a discernibility matrix and extracting certain rules, as in [6]. As mentioned in [30,32], from the different representations of an equivalence relation, Yao first proposed three constructive definitions of Pawlak rough set approximations, i.e., an element-based definition, a granule-based definition and a subsystem-based definition. By replacing the partition with a