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A choice model with imprecise ordinal evaluations ☆

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ABSTRACT

We consider multicriteria choice problems where the actions are evaluated on ordinal criteria and where they can be assessed imprecisely. In order to select the subset of best actions, the pairwise comparisons between the actions on each criterion are modeled by basic belief assignments (BBAs). Dempster's rule of combination is used for the aggregation of the BBAs of each pair of alternatives in order to express a global comparison between them on all the criteria. A model inspired by ELECTRE I is also proposed and illustrated by a pedagogical example.

1. Introduction

Evidence theory [1,2], also called Dempster–Shafer theory or belief functions theory, is a model that deals with imperfection in data and that offers several tools for combining information. This theory has been proposed as a generalization of the subjective probability theory. It has been also the starting point of many models such as the transferable belief model [3,4]. Furthermore, it has been applied in many fields such as pattern classification [5], clustering [6,7], multicriteria decision aid (MCDA) [8–14], etc.

MCDA [15–17] is a discipline that deals with problems involving multiple conflicting criteria. Within this field, authors generally distinguish three main problems: the choice, the ranking and the classification. A choice problem consists in selecting, among a set of actions, a small subset of alternatives considered as the best according to the criteria set. Such problems often arise in applications such as the projects selection problem [18], the suppliers selection problem [19], etc.

Within MCDA, the modeling phase requires the identification of different kinds of data: evaluation table, criteria weights, preference parameters, etc. In most cases, this assessment step cannot be perfectly achieved and therefore imprecise and uncertain data should be considered. In particular, the actions can be evaluated imperfectly on the considered criteria. This can be due for instance to subjective assessment of the actions, to inexact measurements or to the unstable characteristic of some values [20]. In their paper, Ben Amor et al. [21] have considered multicriteria problems where the actions are evaluated on ordinal and cardinal criteria and where the evaluations are imperfect and modeled by probability functions, possibility distributions, fuzzy measures, basic belief assignments (BBA), etc. In order to compare the evaluations, they are transformed at first into functions having similar properties to probability distributions. For instance, the pignistic transformation [22] is used to transform a BBA into a pignistic probability function. This induces of course a loss of information. The stochastic dominance is then applied to compare the functions deduced from the transformation. This can lead to incomparable evaluations. In this paper, we will be interested to choice problems where the alternatives can be evaluated imprecisely on a set of ordinal criteria.

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The pairwise comparison of the evaluations constitutes the distinctive feature of the outranking methods (ELECTRE approaches [23], PROMETHEE [24,25], etc.). When the two evaluations are precise, it is easy to express situations of strict preference or indifference between them. When at least an evaluation is imprecise, it is impossible to express in several cases strict preference or indifference relations. Within evidence theory, the imprecision of the evaluations can be handled using the function "basic belief assignment" (BBA). The comparison of these BBAs is performed using the first belief dominance (FBD) [13,26] or the RBBD approach (RBBD I and RBBD II) [14]. FBD is a generalization of the first stochastic dominance [27] which permits pairwise comparisons between the BBAs. It has been used to model a choice method inspired by ELECTRE I [13]. RBBD allows the comparison and the ranking of the BBAs based on belief distances. It has been applied in a model inspired by the Xu et al.s' method [14] to rank evaluations expressed by BBAs on each criterion. However, the FBD and RBBD I concepts can lead to incomparable BBAs, i.e., to a situation where we cannot be able to express indifference or strict preference between the BBAs. On the contrary, the RBBD II approach leads to comparisons without incomparabilities, but the results induced by this concept can be viewed as excessive because a loss of information can be induced.

In this work, we will propose a choice model inspired by an outranking method called ELECTRE I [28] in a context where the evaluations of the actions can be imprecise. The pairwise comparisons of the actions on each criterion will be modeled by BBAs expressing preference situation(s) between them. The problem of incomparability between the evaluations will be avoided in this model. Moreover, Dempster's rule of combination will be used in the aggregation of these BBAs on all the criteria.

This paper is structured as follows: in Section 2 we introduce the key concepts of evidence theory, then the different cases related to the comparison of precise and imprecise evaluations are presented in Section 3 and the choice model inspired by ELECTRE I is proposed in Section 4. The model is illustrated by a pedagogical example in Section 5.

2. Evidence theory

Evidence theory is a convenient framework for modeling imperfect data and for combining information. In this section, the main concepts of this theory are briefly introduced. The interested reader can refer to [29] for further details and recent state of art surveys.

2.1. Knowledge model

Basically, imperfection in data within evidence theory is modeled using a function called Basic Belief Assignment (BBA). This function is defined on a finite set of mutually exclusive and exhaustive statements called the frame of discernment. Let $\Theta = \{S_1, S_2, \ldots, S_n\}$ be this set and 2^Θ be the powerset of Θ. A BBA [2] is a function m defined from 2^Θ to $[0,1]$ such as $m\{\emptyset\} = 0$ and $\sum_{A\subseteq\Theta} m(A) = 1$. The quantity $m(A)$, called the basic belief mass of subset A, represents the partial belief that A is true, i.e., the belief committed exactly to A. When $m(A) \neq 0$, A is called a focal element or a focal set. Furthermore, a BBA corresponds to a probability function when all its focal elements are singletons. This particular case of BBA is referred as Bayesian BBA.

A BBA can equivalently be represented by its associated belief (or credibility) and plausibility functions [2] defined from 2^Θ to $[0,1]$ respectively as follows:

$$Bel(A) = \sum_{\substack{B\subseteq A\\ B\neq\emptyset}} m(B) \tag{1}$$

$$Pl(A) = \sum_{A\cap B\neq\emptyset} m(B) \tag{2}$$

The quantity $Bel(A)$ measures the total belief that supports completely A whereas the plausibility $Pl(A)$ quantifies the total belief that can potentially be placed in A, i.e., the belief that supports completely and partially A. Of course, $Bel(\emptyset) = Pl(\emptyset) = 0$, $Bel(\Theta) = Pl(\Theta) = 1$ and $Bel(A) \leqslant Pl(A)$ for all $A \subseteq \Theta$. Moreover, these two functions are connected by the relation $Pl(A) = 1 - Bel(\overline{A})$ where $\overline{A}$ is the complement of A for all $A \subseteq \Theta$.

2.2. Combination

The combination constitutes a crucial component of evidence theory. Within this context, several combination rules have been proposed [30]. Among others, we can mention Dempster's rule [2], Yager's rule [31], Dubois and Prade's rule [32], Murphy's rule [33], Yamada's rule [34], the cautious rule [35], etc.

Dempster's rule, also called the normalized conjunctive rule, is the most commonly-used operator in the combination of BBAs provided by independent sources, i.e., distinct BBAs [30]. This rule is defined in the case of two sources as follows:

$$m_{1|2}(A) = (1-k)^{-1}. \sum_{B\cap C=A} m_1(B).m_2(C) \tag{3}$$