International Journal of Approximate Reasoning 55 (2014) 711–738

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Using possibilistic logic for modeling qualitative decision: Answer Set Programming algorithms

Roberto Confalonieri [a,*,1], Henri Prade [b]

[a] Department of Computing, Goldsmiths College London, London SE14 6NW, United Kingdom
[b] Institut de Recherche en Informatique Toulouse (IRIT), Université Paul Sabatier, 118 Route de Narbonne, 31062 Toulouse Cedex 9, France

ARTICLE INFO

Article history:
Received 12 August 2012
Received in revised form 11 November 2013
Accepted 12 November 2013
Available online 13 November 2013

Keywords:
Qualitative decision under uncertainty
Answer set programming
Possibilistic answer set programming

ABSTRACT

A qualitative approach to decision making under uncertainty has been proposed in the setting of possibility theory, which is based on the assumption that levels of certainty and levels of priority (for expressing preferences) are commensurate. In this setting, pessimistic and optimistic decision criteria have been formally justified. This approach has been transposed into possibilistic logic in which the available knowledge is described by formulas which are more or less certainly true and the goals are described in a separate prioritized base. This paper adapts the possibilistic logic handling of qualitative decision making under uncertainty in the Answer Set Programming (ASP) setting. We show how weighted beliefs and prioritized preferences belonging to two separate knowledge bases can be handled in ASP by modeling qualitative decision making in terms of abductive logic programming where (uncertain) knowledge about the world and prioritized preferences are encoded as possibilistic definite logic programs and possibilistic literals respectively. We provide ASP-based and possibilistic ASP-based algorithms for calculating optimal decisions and utility values according to the possibilistic decision criteria. We describe a prototype implementing the algorithms proposed on top of different ASP solvers and we discuss the complexity of the different implementations.

1. Introduction

The capability of computing a decision under uncertainty represents a desirable and essential feature for an intelligent agent that aims at assisting users in making a decision, such as finding the best day in which organizing a meeting or the best offer among several holiday packages. However, the computation of an optimal decision is not an easy task. The information about the world is usually uncertain and the preferences of the users may have different priorities depending on the possible choices available. To compute an optimal decision, a decision maker agent needs to be equipped with a decision making under uncertainty model. A decision making model takes uncertain knowledge (about the world) and users' preferences into account and ranks available decisions according to a decision criterion which combines certainty and preference degrees in a suitable way.

The formalization of decision making under uncertainty (DMU) emanates from economics and game theory with the *expected utility theory* of von Neumann and Morgenstern [43] and with the *subjective expected utility* of Savage [50]. According to these classical decision theories, available decisions are ranked according to an *expected utility function* which combines probability values, encoding the uncertainty, with utility values encoding users' preferences. These approaches require that a probability distribution and an utility function are fully specified and available. In practice, such functions and distributions

* Corresponding author.
E-mail addresses: confalonieri@lsi.upc.edu (R. Confalonieri), prade@irit.fr (H. Prade).
1 This work was initiated when the first author was at Institute de Recherche en Informatique Toulouse (IRIT).

0888-613X/$ – see front matter
http://dx.doi.org/10.1016/j.ijar.2013.11.002

are not always accessible, or are difficult to obtain [21]. For instance, a user may be reluctant to provide numerical utilities for all possible choices, or only a partial description of the uncertainty can be retrieved. Since an expected utility approach needs numerical probabilities and numerical utilities, the automatization of a decision making process rises some practical limitations of classical decision theory approaches regarding the elicitation of these numbers. Moreover, since preferences and knowledge can be human-oriented, a qualitative representation of information can be more adequate than a quantitative one. These limitations have fostered the appearance of the paradigm of *qualitative decision theory* [21].

Qualitative decision theory refers to more than one kind of representation and several qualitative decision making frameworks have been proposed in the literature [6,8,9,25,28,53,54]. Among them, some works consider all-or-nothing notions of utility and plausibility [6], other works use sets of integers to describe imprecise probabilities or utilities [53,54], while other works propose the use of non-classical logics for qualitative decision [8,9,25,28]. Decision making with qualitative preferences and uncertainty has been studied in the setting of possibility theory by assuming a commensurateness hypothesis between the level of certainty and the preferences priority. As in classical utility theory [43,50], pessimistic and optimistic criteria have been proposed and justified on the basis of postulates [28]. This approach has been transposed into possibilistic logic in which the available knowledge is described by formulas which are more or less certainly true and the goals are described in a separate prioritized base.

Although possibilistic logic offers a convenient qualitative framework for modeling decision under uncertainty, the computation of optimal decisions in possibilistic logic has received less attention. In the literature, the only work that addresses the implementation of the possibilistic decision criteria is [24], in which an ATMS-based computation is proposed. Therefore, looking for another implementation of possibilistic qualitative decision making is interesting. To this end, we have looked for an alternative setting on top of which the possibilistic decision making model can be implemented. A paradigm for knowledge representation and computing worthy to be explored is the *Answer Set Programming* (ASP) framework [2].

ASP is a logic programming framework which is considered to be expressive enough to address many knowledge representation problems in Artificial Intelligence (AI). Several efficient ASP solvers have been implemented [32,36,45] and several extensions to deal either with uncertainty [44], or with preferences [13] or with both [16], have been proposed. To this end, it is interesting to look at the ASP setting from a decision making perspective to see whether it can be used to compute optimal decisions according to the possibilistic criteria.

Despite the nice computational features offered by ASP [2], its use for modeling qualitative DMU and for computing optimal decisions has remained almost unexplored. Existing ASP-based methodologies for handling decision making problems [10,37] suggest to model a decision making problem by means of the formalism of Logic Programs with Ordered Disjunction (LPODs) [13]. Although such approaches are enough to cover decisions in completely certain environment, they become less effective when the knowledge is pervaded with uncertainty. Besides, they treat preferences as part of the knowledge. As a consequence, the handling of decision making in ASP differs from the qualitative framework for modeling decision under uncertainty based on possibility theory (see Section 2.1 for a discussion).

In this paper, we present ASP-based algorithms to calculate optimal decisions according to the decision criteria formulated in possibility theory. Although the calculation of optimal decisions in the possibilistic setting was addressed for the first time in terms of an ATMS-based computation [24], this paper offers – to the best of our knowledge – a novel approach for modeling qualitative DMU and for computing the possibilistic criteria in the ASP setting. This is valuable since decision making is a common issue in many applications and ASP solvers exist on top of which our algorithms can be implemented.[2]

According to the approach we present, we translate a decision problem into a problem tractable by an ASP-based and a possibilistic ASP-based computation. First, in Section 3, we formulate qualitative decision making in terms of an LPOD-based construction for abductive logic programming [11], by considering the case in which knowledge is fully certain and preferences are all-or-nothing (Propositions 1–2). This step is necessary to build the general case in which knowledge and preferences are a matter of degrees. Based on that, in Section 4, we show how to handle both a knowledge base pervaded with uncertainty and a prioritized preference base, how to adapt the optimistic and pessimistic decision criteria to the ASP setting and, how to compute both criteria in terms of the LPODs semantics (Propositions 3–4).

Furthermore, we propose a possibilistic ASP-based algorithm to compute the decision criteria by taking the following property of possibilistic logic into account. In [23], it is proven that it is possible to entail a possibilistic formula with certainty α from a possibilistic knowledge base if and only if the same formula can be entailed from the α-cut of the possibilistic knowledge base.[3] By generalizing this property to the logical view of a decision problem, a direct relation between the calculation of an optimal decision according to a classical and a possibilistic logic approach exists. Based on that, in Section 4.3, we present two alternative algorithms to compute an optimal decision and utility values based on the framework of Logic Programs with Possibilistic Ordered Disjunction (LPPODs) [16], a possibilistic extension of LPODs. We show how the computations of optimal decisions and utility values according to LPODs and LPPODs are equivalent (Propositions 7–8). This is an important result since it proves that we can use different semantics, namely the LPODs and possibilistic LPPODs semantics, for computing the possibilistic criteria in a consistent way.

We describe a prototype supporting decision making under uncertainty. The prototype implements the algorithms based on the LPODs and LPPODs semantics. Furthermore, we show how our algorithms can also be generalized to other semantics

[2] In Section 5, we will present a prototype that implements the algorithms proposed in the paper.

[3] Informally, the α-cut of a possibilistic knowledge base is defined as the subset of formulas that are at least α certain without certainty degrees. Possibilistic theories and α-cut are introduced in Section 2.2.