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Multi-adjoint fuzzy rough sets: Definition, properties and attribute selection

Chris Cornelis [a,c,1], Jesús Medina [b,2,*], Nele Verbiest [c]

[a] Department of Computer Science and Artificial Intelligence, University of Granada, Spain
[b] Department of Mathematics, University of Cádiz, Spain
[c] Department of Applied Mathematics, Computer Science and Statistics, Ghent University, Gent, Belgium

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ABSTRACT

This paper introduces a flexible extension of rough set theory: multi-adjoint fuzzy rough sets, in which a family of adjoint pairs are considered to compute the lower and upper approximations. This new setting increases the number of applications in which rough set theory can be used. An important feature of the presented framework is that the user may represent explicit preferences among the objects in a decision system, by associating a particular adjoint triple with any pair of objects.

Moreover, we verify mathematical properties of the model, study its relationships to multi-adjoint property-oriented concept lattices and discuss attribute selection in this framework.

1. Introduction

Pawlak proposed rough set theory [23] in the 1980s as a formal tool for modeling and processing incomplete information in information systems.

On the other hand, formal concept analysis, introduced by Wille in the decade of 1980 [28], arose as another mathematical theory for qualitative data analysis and, currently, has become an interesting research topic both with regard to its mathematical foundations [16,25] and with regard to its multiple applications [5,6].

These mathematical theories have been related in several papers [7,8,14,15,18,19]. In particular, property-oriented concept lattices [1,4,10] and object-oriented concept lattices [29] were introduced in order to extend formal concept lattices [9], with constructs from rough set theory; notably, they invoke the lower and upper approximation operators, which are often referred to in this research field as necessity and possibility operators, respectively.

More recently, multi-adjoint property-oriented and object-oriented concept lattices were studied [17,18], with the aim of introducing adjoint triples of fuzzy logic operators (in particular, a conjunctor and its two residuated implications) to define "soft" extensions of the necessity and possibility operators. This is similar to what happens in fuzzy rough set theory, where a t-norm and fuzzy implication are used in order to extend the classical rough lower and upper approximation operators.

* Corresponding author.
E-mail addresses: chriscornelis@ugr.es (C. Cornelis), jesus.medina@uca.es (J. Medina), Nele.Verbiest@UGent.be (N. Verbiest).

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However, the multi-adjoint paradigm goes much further in the sense that it allows us to use several adjoint triples, in order to be able to express preferences among objects or properties.

In this paper, the latter characteristic of multi-adjoint property-oriented concept lattices is introduced into the framework of fuzzy rough sets, that is to say, we propose a multi-adjoint fuzzy rough set model in which the lower and upper approximation operators are constructed using several adjoint triples. This allows us to introduce explicit preferences among the objects, by associating a particular adjoint triple with any pair of objects in a decision system.

We study various properties of the model and focus in particular on attribute selection. From the perspectives of both concept lattices and rough sets, attribute selection is an important step in reducing the computational complexity. Recently, Wang and Zhang related attribute selection in property-oriented and object-oriented concept lattices [27]. Moreover, in [22], two kinds of reduction methods have been proposed and the relationship with attribute selection in rough set theory is discussed in detail.

The remainder of this paper is structured as follows. In Section 2, we recall preliminaries from rough sets, fuzzy rough sets and multi-adjoint property-oriented concept lattices. Next, in Section 3, we define multi-adjoint fuzzy rough sets and investigate their main properties. We also define a general positive region to focus on the decision attribute and on the attribute selection based on multivalued measures. In Section 4, the notions of L-valued measure, m, and fuzzy m-decision reduct are included, and specific measures are studied based on the positive region notion and on a fuzzy discernibility function which generalize the ones given in [3]. Moreover, we introduce a relation between them. Finally, in Section 5, we conclude the paper.

2. Preliminaries

2.1. Rough set theory

In the framework of rough set theory, data is represented as an *information system* $(X, \mathcal{A})$, where $X = \{x_1, \ldots, x_n\}$ and $\mathcal{A} = \{a_1, \ldots, a_m\}$ are finite, non-empty sets of objects and attributes, respectively. Each a in $\mathcal{A}$ corresponds to a mapping $\bar{a} : X \to V_a$, where V_a is the value set of a over X. For every subset B of $\mathcal{A}$, the B-indiscernibility relation[3] R_B is defined as the equivalence relation

$$R_B = \left\{(x, y) \in X \times X \;\middle|\; \text{for all } a \in B,\ \bar{a}(x) = \bar{a}(y)\right\} \tag{1}$$

Given $A \subseteq X$, its lower and upper approximation w.r.t. B are defined by

$$R_B{\downarrow}A = \left\{x \in X \;\middle|\; [x]_{R_B} \subseteq A\right\} \tag{2}$$

$$R_B{\uparrow}A = \left\{x \in X \;\middle|\; [x]_{R_B} \cap A \neq \emptyset\right\} \tag{3}$$

A *decision system* $(X, \mathcal{A} \cup \{d\})$ is a special kind of information system, in which $d \notin \mathcal{A}$ is called the decision attribute, and its equivalence classes $[x]_{R_d}$ are called decision classes. Given $B \subseteq \mathcal{A}$, the B-positive region, POS_B, and the degree of dependency of d on B, γ_B, are defined as

$$POS_B = \bigcup_{x \in X} R_B{\downarrow}[x]_{R_d} \tag{4}$$

$$\gamma_B = \frac{|POS_B|}{|X|} \tag{5}$$

$(X, \mathcal{A} \cup \{d\})$ is called *consistent* if $\gamma_{\mathcal{A}} = 1$. A subset B of $\mathcal{A}$ is called a *decision reduct* if it satisfies $POS_B = POS_{\mathcal{A}}$ and there exists no proper subset B' of B such that $POS_{B'} = POS_{\mathcal{A}}$.

A well-known approach to generate all reducts of a decision system is based on its discernibility matrix and function [26]. The discernibility matrix of $(X, \mathcal{A} \cup \{d\})$ is the $n \times n$ matrix O, defined by, for i and j in $\{1, \ldots, n\}$,

$$O_{ij} = \begin{cases} \emptyset & \text{if } d(x_i) = d(x_j) \\ \{a \in \mathcal{A} \mid \bar{a}(x_i) \neq \bar{a}(x_j)\} & \text{otherwise} \end{cases} \tag{6}$$

The *discernibility function* of $(X, \mathcal{A} \cup \{d\})$ is the map $f : \{0, 1\}^m \to \{0, 1\}$, defined by

$$f\left(a_1^*, \ldots, a_m^*\right) = \bigwedge\left\{\bigvee O_{ij}^* \;\middle|\; 1 \leqslant i < j \leqslant n \text{ and } O_{ij} \neq \emptyset\right\} \tag{7}$$

in which $O_{ij}^* = \{a^* \mid a \in O_{ij}\}$. The boolean variables $a_1^*, \ldots, a_m^*$ correspond to the attributes from $\mathcal{A}$. It can be shown that the prime implicants of f constitute exactly all decision reducts of $(X, \mathcal{A} \cup \{d\})$.

[3] When $B = \{a\}$, i.e., B is a singleton, we will write R_a instead of $R_{\{a\}}$.