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Duality, conjugacy and adjointness of approximation operators in covering-based rough sets

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ABSTRACT

Many different proposals exist for the definition of lower and upper approximation operators in covering-based rough sets. In this paper, we establish relationships between the most commonly used operators, using especially concepts of duality, conjugacy and adjointness (also referred to as Galois connection). We highlight the importance of the adjointness condition as a way to provide a meaningful link, aside from duality, between a pair of approximation operators. Moreover, we show that a pair of a lower and an upper approximation operator can be dual and adjoint at the same time if and only if the upper approximation is self-conjugate, and we relate this result to a similar characterization obtained for the generalized rough set model based on a binary relation.

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1. Introduction

Rough set theory, proposed by Pawlak [16] in 1982, is a prominent tool for dealing with uncertainty and incompleteness in information systems. It revolves around the notion of discernibility of objects, which classically is represented by means of an equivalence relation, or equivalently a partition of the set of objects. The theory has been generalized from different perspectives. One generalization of rough sets is to replace the equivalence relation by a general binary relation. In this case, the binary relation determines collections of sets that no longer form a partition of U. This generalization has been used in applications with incomplete information systems and tables with continuous attributes [9,10,27]. A second generalization is to replace the partition obtained by the equivalence relation with a covering; i.e., a collection of nonempty sets with union equal to U. There are many works in these two directions and some connections between the two generalizations have been established, for example in [27,31,41,40,38].

In our work, we focus on covering-based approaches. Recently, Yao and Yao [32] introduced a general framework for the study of dual pairs of covering-based approximation operators, distinguishing between element-, granule- and subsystem-based definitions. They emphasized the crucial importance of duality of the lower and upper approximations, stating that without this condition one has no reason to investigate them jointly as a pair. Nevertheless, a lot of other approximation pairs outside this framework have been studied in literature; for instance, in [25], Yang and Li present a summary of seven non-dual pairs of approximation operators used by Żakowski [33], Pomykala [17], Tsang et al. [18], Zhu [40], Zhu and Wang [42], Xu and Wang [23]. On the other hand, in a recent paper, Ciucci et al. [4] study opposition diagrams for dual and non-dual approximations in rough set theory, indicating the relevance of both.

In this paper, we draw attention to another important property of classical rough sets, namely the fact that the lower approximation $\underline{apr}$ and the upper approximation $\overline{apr}$ form a Galois connection, or put differently, that they make up an

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adjoint pair. Galois connections have been used in various areas of mathematics and theoretical computer science as a way of providing a strong link between (morphisms on) ordered structures [1,15]. In the context of rough sets, the adjointness property guarantees for example that the fix points of $\underline{apr}$ and $\overline{apr}$ coincide, in other words, $\underline{apr}(A) = A$ iff $\overline{apr}(A) = A$. This is particularly important when considering the notion of definable sets, i.e., subsets of the universe for which the approximations equal the original set. Moreover, if an upper approximation $\overline{apr}$ has an adjoint $\underline{apr}$, then it is unique, and vice versa a lower approximation $\underline{apr}$ can have at most one co-adjoint, so the Galois connection, like duality, provides a one-to-one connection between approximation operators.

The adjointness property also plays an important role in the definition of logical systems derived from rough sets, in particular modal logics, since lower approximation is related with the necessity operator and upper approximation with the possibility operator. For example, Järvinen et al. describe in [12] an information logic for rough sets using the Galois connection, while Yao shows in [30] that adjointness is necessary for modal logic type B. Furthermore, a Galois connection also holds between the set theoretic operators used in Formal Concept Analysis, which bears a close relationship to rough set theory [28].

The objective of this paper is twofold: first, we want to establish relationships between the various definitions of approximation operators in the covering-based rough set model; for this, we will be using some lattice concepts for rough sets, such as meet and join preserving functions, conjugates, duals and Galois connections. Secondly, we want to evaluate the existing proposals with respect to the adjointness condition, providing in particular a characterization of approximation operators pairs that are both dual and adjoint. In this way, we hope to provide a clear cut roadmap for the covering-based rough set landscape, pinpointing the most useful operators among the many that have been proposed in the literature and guiding future research directions. In particular, we believe our results are helpful to select appropriate approximation operators in typical rough set applications such as attribute selection and classification; unlike in the classical case, there are many possibilities to define the approximations, and the particular choice is likely to affect the quality of these applications.

The remainder of this paper is organized as follows. Section 2 presents preliminary concepts about rough sets and lower and upper approximations in covering-based rough sets, as well as the necessary lattice concepts about duality, conjugacy and adjointness. In Section 3, we present equivalences and relationships between various approximation operators, evaluate which of them satisfy adjointness, and end with the characterization theorem for dual and adjoint pairs. We also relate this characterization with previous results in generalized rough set theory based on a binary relation. Finally, Section 4 presents some conclusions and outlines future work.

2. Preliminaries

Throughout this paper, we assume that U is a finite and nonempty set. $\mathscr{P}(U)$ represents the collection of subsets of U.

2.1. Three definitions of Pawlak rough sets approximations

In Pawlak's rough set model, an approximation space is an ordered pair $apr = (U, E)$, where E is an equivalence relation on U. According to [29,32], there are three different, but equivalent ways to define lower and upper approximation operators: element-based definition, granule-based definition and subsystem-based definition. For each $A \subseteq U$, the lower and upper approximations are defined by:

Element-based definition

$$\underline{apr}(A) = \{x \in U\colon [x]_E \subseteq A\} \tag{1}$$

$$\overline{apr}(A) = \{x \in U\colon [x]_E \cap A \neq \emptyset\} \tag{2}$$

Granule-based definition

$$\underline{apr}(A) = \bigcup\{[x]_E \in U/E\colon [x]_E \subseteq A\} \tag{3}$$

$$\overline{apr}(A) = \bigcup\{[x]_E \in U/E\colon [x]_E \cap A \neq \emptyset\} \tag{4}$$

Subsystem-based definition

$$\underline{apr}(A) = \bigcup\{X \in \sigma(U/E)\colon X \subseteq A\} \tag{5}$$

$$\overline{apr}(A) = \bigcap\{X \in \sigma(U/E)\colon X \supseteq A\} \tag{6}$$

where $\sigma(U/E)$ is the σ algebra that is obtained from the equivalence classes U/E, by adding the empty set and making it closed under set unions.

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