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# Dempster–Shafer fusion of evidential pairwise Markov fields

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ABSTRACT

Hidden Markov fields (HMFs) have been successfully used in many areas to take spatial information into account. In such models, the hidden process of interest $X$ is a Markov field, that is to be estimated from an observable process $Y$. The possibility of such estimation is due to the fact that the conditional distribution of the hidden process with respect to the observed one remains Markovian. The latter property remains valid when the pairwise process $(X, Y)$ is Markov and such models, called pairwise Markov fields (PMFs), have been shown to offer larger modeling capabilities while exhibiting similar processing cost. Further extensions lead to a family of more general models called triplet Markov fields (TMFs) in which the triplet $(U, X, Y)$ is Markov where $U$ is an underlying process that may have different meanings according to the application. A link has also been established between these models and the theory of evidence, opening new possibilities of achieving Dempster–Shafer fusion in Markov fields context. The aim of this paper is to propose a unifying general formalism allowing all conventional modeling and processing possibilities regarding information imprecision, sensor unreliability and data fusion in Markov fields context. The generality of the proposed formalism is shown theoretically through some illustrative examples dealing with image segmentation, and experimentally on hand-drawn and SAR images.

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## 1. Introduction

Let $S$ be a finite set, with $Card(S) = N$, and let $(Y_s)_{s \in S}$ and $(X_s)_{s \in S}$ be two collections of random variables, which will be called "random fields". We assume that $Y$ is observable with each $Y_s$ taking its values in $\mathbb{R}$ (or $\mathbb{R}^d$) whereas $X$ is hidden with each $X_s$ taking its values from a finite set of "classes" or "labels". Such situation occurs in image segmentation problem, which will be used in this paper as an illustrative frame. Realizations of such random fields will be denoted using lowercase letters. We deal with the problem of the estimation of $X = x$ from $Y = y$. Such estimation subsumes the distribution of $(X, Y)$ to be beforehand defined. One classic way to do so is to define, on one hand, the distribution of $X$, usually called "prior" distribution, and on the other hand, the distribution of $Y$ conditional on $X$, usually called "noise" distribution. When the prior distribution is Markov, such models are called "hidden Markov fields" (HMFs). These models are of interest as they allow one to find optimal Bayesian solutions, and are successfully used for about forty years [1,2]. HMFs can be extended to "pairwise Markov fields" (PMFs), in which one directly assumes Markovianity of the pair $(X, Y)$ [3], and PMFs have been extended to "triplet Markov fields" (TMFs), in which a third finite discrete valued random field $(U_s)_{s \in S}$ is introduced and

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the triplet $(U, X, Y)$ is assumed Markov [4,5]. Finally, TMFs have been extended to "conditional" TMFs (CTMFs), in which one assumes the Markovianity of $(U, X)$ conditional on $Y$ [6]. Likely to HMFs, Bayesian processing can be performed in PMFs, TMFs, and CTMFs as well.

On the other hand, Dempster–Shafer fusion (DS fusion) performed in the frame of "theory of evidence" (TE) allows one to fuse information of different natures [7–12]. The core point used in the paper is that DS fusion can be seen as an extension of the probabilistic computation of the "*a posteriori*" distribution needed in Bayesian processing mentioned above, and thus this processing can be used in a more general setting. Such ideas have already been applied in some special situations. In particular, it has been shown that using DS fusion and Markov field models simultaneously can be of interest [13–17]. Some extensions of the standard HMFs using the theory of evidence are proposed to segment images in [13]. The problem of data fusion of radar and optical images with cloud cover is considered in [14]. Tupin et al. use DS fusion of several structure detectors for automatic interpretation of SAR images [15]. Notice that theory of evidence has also been used within hidden Markov chains for image modeling-related problems. In [18], hidden evidential Markov chains are applied for nonstationary image segmentation. In [19], DS fusion is used to fuse multisensor data in nonstationary Markovian context. Other applications of evidential Markov models include data fusion and image classification [20], power quality disturbance classification [21], particle filtering [22], prognostics [23] and fault diagnosis [24]. Ramasso and Denoeux use belief functions to introduce partial knowledge about hidden states of an HMM [25]. In [16], authors use evidential reasoning to relax Bayesian decisions given by a Markovian classification. The approach is applied to noisy images classification. In [26], a method is developed to prevent hazardous accidents due to operators' action slip in their use of a Skill-Assist. In [27], a second-order evidential Markov model is introduced. Finally, let us mention that the use of imprecise probabilities [28–30] to extend the above models may also be investigated.

The purpose of this paper is to propose a very general family of models providing an original unifying formalism, allowing different known modeling and processing possibilities regarding information imprecision, sensor unreliability and data fusion in the Markov fields context. More precisely:

(i) the proposed family is closed with respect to DS fusion;
(ii) it contains new "conditional evidential Markov models";
(iii) it contains new nonstationary evidential Markov models.

As will be seen, the first point is the core one as it will allow one to perform DS fusion in a very workable manner, by simply adding the corresponding Markov energies. This is of interest because while trying to perform DS fusion in Markov fields in a classic manner one arrives to a non-tractable sum.

Besides this greater generality and the theoretical interest of the proposed extensions, let us mention a specific advantage of a particular new model with respect to the model proposed in [13]. In the case where the noise is complex and its form is not known, the new model makes it possible to approximate the unknown forms of noise through Gaussian mixtures. This may be of practical interest, as shown through some experiments provided in section 4. Indeed, this is all the more of a practical use since parameters can be estimated with "iterative conditional estimation" (ICE) method [4,5], and thus, segmentation can be achieved in the unsupervised context.

Let us notice that a great deal of papers have been published on HMFs, and the same is true on DS information fusion. However, papers dealing simultaneously with both of these topics are relatively rare. Thus this paper is also intended to readers who are used with one of these theories, and not necessarily with the other one. This is why there are some developments, and numerous examples, which could appear as obvious for some readers but of interest for others. Let us mention that an analogous general formalism has been proposed in the frame of Markov chains in [31].

The remainder of this paper is organized as follows: section 2 recalls different Markov field models, the theory of evidence, and its use within particular Markov models. Section 3 describes the proposed evidential pairwise Markov field and its associated theory. Experimental results obtained on hand-drawn and SAR images are provided in section 4. Concluding remarks and future directions are given in section 5.

## 2. Theory of evidence and hidden Markov fields

This section contains four paragraphs. In the first one, we briefly recall the basics of the theory of evidence. As it also addresses readers possibly ignoring TE, the theory is presented in a simple classic format. The second paragraph is devoted to illustrate the interest of TE in Bayesian classification. The examples presented are rather simple for TE experts; however, they can be of immediate interest to readers familiar with Bayesian image segmentation, specifying different situations they may be faced with. Classic hidden Markov fields, pairwise Markov fields, and triplet Markov fields are recalled in paragraph 3. Finally, in the last fourth paragraph we recall how a simple HMF can be extended to an evidential Markov field using triplet Markov fields.

### 2.1. Theory of evidence

Let $\Omega = \{\omega_1, \ldots, \omega_K\}$, and let $P(\Omega) = \{A_1, \ldots, A_q\}$ be its associated powerset, with $q = 2^K$. A function $M$ from $P(\Omega)$ to $[0, 1]$ is called a "basic belief assignment" (*bba*) if $M(\emptyset) = 0$ and $\sum_{A \in P(\Omega)} M(A) = 1$. A *bba* $M$ defines then a "plausibility"

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