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Variational preferences and equilibria in games under ambiguous belief correspondences ☆

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ABSTRACT

Variational preferences have been introduced to study the robustness of macroeconomic models with respect to ambiguity. The Decision Theory literature has shown that this family of preferences provides a tractable and flexible tool in order to deal with this kind of uncertainty in general settings. In fact, variational preferences are equipped with an accurate and explicit parametrization of the decision maker's attitude towards ambiguity, which is indeed represented by a cost function of the probabilities (called *index of ambiguity aversion*). In the present work, we study the effect of variational preferences in strategic form games under ambiguity in which players' uncertainty is expressed entirely in the space of lotteries over consequences by *belief correspondences* of the strategy profile chosen by the agents. We focus on primary theoretical issues related to this model that constitute a required background for applications or numerical methods. First, we give a general equilibrium existence result that we apply to a particular model in which belief correspondences depend on the equilibria of specific subgames. Other numerical examples are presented to show the model applicability. Finally, we look at the consequences of parameter changes on equilibrium predictions and study the limit behavior of equilibria under perturbations on the index of ambiguity aversion and belief correspondences. All the results are sufficiently general to be a useful tool in any interdisciplinary problem in which strategic interaction is affected by ambiguity.

1. Introduction

It is well known that strategic decisions under uncertainty often involve probabilities that are only partially specified. In this paper, we look at strategic form games under ambiguity in which uncertainty cannot always be described by *well defined* probabilities over the set of outcomes of the game but it can instead be represented by sets of probability distributions. Sets of probability distributions have been used to represent uncertainty in classical decision theory models under ambiguity[1]; in those models, the source of uncertainty is described by an underlying state space and ambiguous beliefs are represented

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[1] Sets of probability distributions are also recognized as one of the possible representation of *imprecise probabilities* (see for instance in [31]).

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by fixed subsets of priors (probability distributions) over this set of states.[2] The decision maker's action set is a subset of *acts* which are functions from the state space to a space of *consequences* and he is endowed with a utility function which gives a numerical outcome for every possible consequence, that is for every action choice and realization of the state. Recent papers have also investigated this kind of ambiguity in strategic interactions: Kajii and Ui [16] first investigate the effects of uncertainty aversion in incomplete information games with multiple priors. Bade [5] considers games à la Aumann under more general preferences. Finally, Azrieli and Teper [4] characterize equilibrium existence in terms of the preferences of the players.[3]

There is a different strand of research in decision theory which shows that an agent, facing ambiguity, is not able to understand what the relevant states are and therefore the information available can be expressed entirely in the space of probabilities (lotteries) over consequences (see [1], [25] and [30]). Despite the approach *without the state space* and the classical *multiple prior* approach can be reconciled by a generalized form of probabilistic sophistication where an ambiguous act is evaluated by its induced set of distributions over consequences (see [1] and [25]), game theory provides further evidence that ambiguity cannot always be reconducted to the classical approach with a state space and multiple priors. In fact, the literature on ambiguous games (see for instance [11,21,18,12,23,19,26]) has shown that in a game there is a specific source of ambiguity since players may have ambiguous beliefs about opponents' strategy choices. There is no evidence in the literature showing that this kind of ambiguity can be properly reconducted to incomplete information games à la Harsanyi (that is, with state space) under multiple priors which, in turn, must be generalized in order to encompass this specific game theoretical issue. In previous papers, we introduced and studied the (so-called) model of *game under ambiguous belief correspondences*[4] which provides a rather general tool to study ambiguity in games. The key point of our approach is that, for every player, ambiguity is directly represented by a belief correspondence which maps the set of strategy profiles to the set of all subsets of probability distributions over the outcomes of the game. For each player and for every given strategy profile, the belief correspondence gives the set of probability distributions over the possible outcomes of the game that the corresponding player perceives to be feasible and consistent with the actual strategy profile. On the one hand, belief correspondences might represent objective (exogenous) ambiguity as done in [1] and [25]; on the other hand, it turns out (see the examples in [10]) that many existing models of ambiguous game have an equivalent formulation in terms of belief correspondences. For example, a notion of equilibrium in incomplete information games with multiple priors and the partially specified equilibrium concept by Lehrer [19] can both be regarded as particular cases of our notion of *equilibrium under ambiguous belief correspondences*.

In our previous works, we considered players endowed only with the classical maxmin preferences.[5] In this paper we study a generalization of our equilibrium concept in which we relax the assumption imposed on the ambiguity attitudes of the players. More precisely we consider players endowed with *variational preferences* as introduced and axiomatized by Maccheroni, Marinacci and Rustichini [22]. This class of preferences generalizes maxmin preferences and embodies the approach of the works of Hansen and Sargent [15] on uncertainty in macroeconomics since their model provides a particular class of variational preferences as well. Under variational preferences, players evaluate any ambiguous belief by the worst possible value (given the set of probability distributions) assumed by the sum of the expected utility with a nonnegative function of the probabilities called *index of ambiguity aversion*. This index plays a very important role as a measure of ambiguity aversion. For instance, maximal ambiguity aversion corresponds to index ambiguity aversion identically equal to zero and gives back maxmin preferences. While, minimal ambiguity aversion corresponds to ambiguity neutrality and gives back subjective utility preferences. Given the flexibility of the variational preferences model, it seems interesting and natural to look at this kind of preferences in games. In [9] we extend the Kajii and Ui's notion of mixed equilibrium by allowing for variational preferences and we investigate the issue of the existence and stability of equilibria.[6] In this paper we look at the equilibria in games under variational preferences in the more complex model with ambiguous belief correspondences; first, we present an existence theorem and then we show in an illustrative example (in which beliefs over opponents' strategy profile are given by the set of Nash equilibria of specific subgames) that the assumptions in the main existence theorem can be easily obtained in specific applications.

The last section is devoted to the issue of stability of equilibria. The problem of the limit behavior of the equilibria in games has been extensively studied in the literature (see, for instance, [13] for the standard problem, [24,32] for recent results under relaxed or different assumptions and references). The question whether the limit property extends to the equilibrium concepts in ambiguous games has been studied in [9] for the equilibria in the Kajii and Ui model under variational preferences, in [10] for equilibria under ambiguous belief correspondences and maxmin preferences, in [29] for an equilibrium notion in ambiguous games which relies on the Beweley unanimity rule. Maccheroni, Marinacci and Rustichini [22] raise the question of the limit behavior of variational preferences; they show that variational preferences become more

[2] This is known as the *multiple prior* approach firstly studied in [14].

[3] Similar approaches and applications can be found also in [33] and [34].

[4] De Marco and Romaniello [8] present the general model, an existence theorem and many motivating examples. Stability of the equilibria is studied in [10]. An application to coalition formation is the subject of De Marco and Romaniello [7].

[5] Indeed, we considered also their counterpart: maxmax preferences.

[6] The Kajii and Ui's mixed equilibrium is an *interim* equilibrium concept, meaning that it is dynamic equilibrium concept, in a sense. So it cannot be reconducted directly to our concept of equilibrium under belief correspondences. Nevertheless, the corresponding ex-ante equilibrium concept has an equivalent formulation in terms of belief correspondences.