International Journal of Approximate Reasoning 55 (2014) 1404–1418

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

On multi-granulation covering rough sets

Caihui Liu [a,b,*], Duoqian Miao [b], Jin Qian [c]

[a] *Department of Mathematics and Computer Science, Gannan Normal University, 341000 Ganzhou, China*
[b] *Department of Computer Science and Technology, Tongji University, 201804 Shanghai, China*
[c] *College of Computer Engineering, Jiangsu University of Technology, 213015 Changzhou, China*

ARTICLE INFO

Article history:
Received 4 September 2012
Received in revised form 25 October 2013
Accepted 7 January 2014
Available online 21 January 2014

Keywords:
Covering
Lower and upper approximations
Multi-granulation
Reduct
Rough sets

ABSTRACT

Recently, much attention has been given to multi-granulation rough sets (MGRS) and different kinds of multi-granulation rough set models have been developed from various viewpoints. In this paper, we propose four types of multi-granulation covering rough set (MGCRS) models under covering approximation space, where a target concept is approximated by employing the maximal or minimal descriptors of objects in a given universe of discourse U. And then, we investigate a number of basic properties of the four types of MGCRS models, and discuss the relationships and differences among the classical MGRS model and our MGCRS models. Moreover, the conditions for two distinct MGCRS models which produce identical lower and upper approximations of a target concept in a covering approximation space are also studied. Finally, the relationships among the four types of MGCRS models are explored. We find that for any subset $X \subseteq U$, the lower approximations of X and the upper approximations of X under the four types of MGCRS models can construct a lattice, if we consider the binary relation of inclusion.

1. Introduction

To enlarge the usage scope of the Pawlak's rough set model [1], many meaningful extensions have been proposed for various requirements in the past decades, such as the graded rough sets [2], arbitrary binary relations based rough sets [9,29], tolerance or similarity relations based rough sets [30–32], rough fuzzy sets and fuzzy rough sets [33], and variable precision rough sets [34,35], etc. As one of the extension models, covering rough sets, which was first proposed by Zakowski [5], has attracted much attention and induced lots of interesting results [6–9,36,37,40–42]. Recently, Yao et al. [4] proposed a framework for the study of covering based rough set approximations, which enables us to reproduce many existing approximation operators and introduce some new approximation operators.

In the viewpoint of Granular Computing [3,10], Pawlak's rough set model and most of its extensions are constructed based on only one granular structure, which is induced by a binary relation (a partition or a covering). Thus, one may call those models the single-granulation rough sets. As Qian et al. [12] have mentioned that, in many cases, a target concept is needed to describe concurrently from some independent environments, that is, multi-granulation spaces are needed. Therefore, Qian and Liang [11,12] introduced the concept of multi-granulation rough sets (MGRS), where the approximations of a set of objects are defined by using multi-equivalence relations. The main difference between single-granulation rough sets and multi-granulation ones lies in that the approximations of a target concept in multi-granulation rough sets are constructed by using multi-distinct sets of information granules. When two attribute subsets in an information system contradict each other or possess an inconsistent relationship, MGRS will show its advantages for knowledge discovery [12].

* Corresponding author at: Department of Mathematics and Computer Science, Gannan Normal University, 341000 Ganzhou, China.
E-mail addresses: liu_caihui@163.com (C. Liu), miaoduoqian@163.com (D. Miao), qjqjlqyf@163.com (J. Qian).

0888-613X/$ – see front matter
http://dx.doi.org/10.1016/j.ijar.2014.01.002

By now, much attention has been given to the multi-granulation rough sets. For instance, Qian et al. [13] presented a multi-granulation rough set model based on multiple tolerance relations in incomplete information systems. In [14], Qian et al. introduced the concept of positive approximation. The positive approximation can be regarded as a special kind of multi-granulation rough set model, in which the granulation structure of a rough set is characterized by using a granulation order, that is, the granulation structure is characterized by a sequence of attribute sets with granulations from coarse to fine. Based on the positive approximation, Qian et al. developed a common accelerator for improving the time efficiency of a heuristic attribute reduction, which provides a vehicle of making algorithms of rough set based feature selection techniques faster. Based on Ref. [13], Yang et al. [15] further discussed the properties of multi-granulation rough sets in incomplete information systems. Moreover, Yang et al. [16] and Xu et al. [21,24,26] constructed multi-granulation rough sets based on the fuzzy approximation space. Yang et al. [17] discussed the hierarchical structures of multi-granulation rough sets and She et al. [28] investigated the topological and lattice-theoretic properties of multi-granulation rough sets. Liang et al. [39] introduced an efficient rough feature selection algorithm with a multi-granulation view. Liu et al. [18,19] introduced the concepts of multi-granulation covering rough sets and multi-granulation covering fuzzy rough sets in a covering approximation space. Lin et al. [20] discussed two kinds of neighborhood-based multi-granulation rough sets and three kinds of covering based multi-granulation rough sets [43]. Xu et al. [22,25,27] constructed three kinds of multi-granulation rough sets based on the tolerance, ordered and generalized relations respectively. Qian et al. [44] developed the multi-granulation decision-theoretic rough set and proved that many existing multi-granulation rough set models can be derived from the multi-granulation decision-theoretic rough set framework.

As an extension of our previous work [18], the purpose of this paper is to further generalize the classical multi-granulation rough sets to covering environment. Through combining the multi-granulation rough sets with covering rough sets, we aim to solve the limitation of classical multi-granulation rough sets induced by equivalence relations. To define the approximations of a target concept in the multi-granulation covering environment, we respectively employ the maximal and minimal descriptors of objects in a given universe of discourse U. For any subset $X \subseteq U$ and object $x \in U$, we first respectively define the type-1 and type-2 approximations (including low and upper approximations) of X by using the intersection and union of all elements in the minimal descriptor of x. Second, the type-3 and type-4 approximations of X are respectively defined based on the intersection and union of all elements in the maximal descriptor of x. Correspondingly, we can obtain four types of multi-granulation covering rough set (MGCRS) models under a covering approximation space. The basic properties of the four types of MGCRS models, and the relationships among the classical MGRS model and our MGCRS models are discussed. Moreover, the conditions for two distinct MGCRS models which produce identical lower and upper approximations of a target concept in a covering approximation space are also studied. Finally, the relationships among the four types of MGCRS models are explored.

The remainder of this paper is organized as follows. The next section deals with some preliminary concepts and properties regarding the Pawlak's rough sets, covering rough sets and multi-granulation rough sets. In Section 3, we introduce four types of multi-granulation covering rough set models and investigate the basic properties of them. Moreover, based on the concept of reduct of a covering, we give the sufficient conditions for two different MGCRS models to produce identical lower and upper approximations of a target concept. In Section 4, we investigate the relationships among the four types of MGCRS models. Finally, Section 5 concludes the paper.

2. Preliminaries

In this section, we review some basic concepts about the Pawlak's rough sets, covering rough sets and multi-granulation rough sets. The detailed descriptions can be found in [1,2,5,6,11,12].

2.1. The Pawlak's rough sets

Let U be a universe of discourse, and R an equivalence relation on U. R partitions the universe U into disjoint subsets, each subset is called an equivalence class or equivalence granule, the family of all equivalence classes is denoted by U/R. For any $X \subseteq U$, one can describe X by a pair of lower and upper approximations defined as follows.

$$\underline{R}(X) = \{x \in U \mid [x]_R \subseteq X\};$$

$$\overline{R}(X) = \{x \in U \mid [x]_R \cap X \neq \emptyset\}.$$

$\underline{R}(X)$ is called the lower approximation of X, which is the union of all the equivalence classes which are subsets of X, and $\overline{R}(X)$ is called the upper approximation of X, which is the union of all equivalence classes which have non-empty intersection with X. Then $(\underline{R}(X), \overline{R}(X))$ is called the rough sets of X.

Let $\sim X = U - X$, we have the following basic properties of Pawlak's rough sets.