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Evaluation of voltage and current profiles and Joule losses for a half-wavelength power transmission line

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ABSTRACT

Half-wavelength lines (HWLLs) have been studied as an option to be applied to power transmission regarding line lengths around 2500 km for a 60 Hz frequency. The main advantage of this type of transmission line, which has not yet been put in commercial operation in any country, is the elimination of reactive compensation, which contributes to cost reduction if compared to a conventional *ac* transmission. This paper presents a demonstration of voltage and current magnitude profiles along the line, using phasors and graphic calculations. In addition, an algebraic expression for Joule losses is proposed, to be used in economical evaluation of a HWLL solution.

Introduction

Half-wavelength transmission has been studied as an alternative to be applied to very long distance power transmission. It can be described as a conventional *ac* transmission line, slightly longer than 2500 km (for a 60 Hz nominal frequency), with no reactive compensators, as series capacitors or shunt reactors, which represents a reduction of the system cost.

Application of half-wavelength properties to power transmission was first addressed by a paper from Russia in 1940 [1]. In the 1960s, two articles from USA [2] and India [3] returned to this topic. In [2] it is proposed three half-wavelength point-to-point power transmission system designs in order to mitigate operational and design problems associated to traditional long *ac* and *dc* transmission lines. Analysis of power flow, short-circuits, and stability were conducted in order to evaluate the proposed systems. In [3], it was studied some operational problems as: loading of the line and synchronization of the half-wave system.

In [4], it was proposed a model for numerical simulation of corona losses at high electric gradients. It was shown that corona losses drastically limits the sustainable overvoltage and place a limit on transmissible power. Paper [5] discussed and analyzed some aspects regarding to feasibility of single-pole high-speed reclosure, temporary overvoltage limitation, and parallel operation of the half-wavelength transmission lines.

A solution to power transmission through very long distances is of great interest to countries with large territorial dimensions. In recent years, studies were conducted in Brazil to establish technical and economic conditions in which half-wavelength would be a viable solution [6–10]. It is noteworthy that there is no half-wavelength power transmission system in commercial operation in the world.

The technical and economical discussion about half-wavelength transmission is based in the particular behavior of the voltage and current phasors along the line. All previous papers about this subject obtained the voltage and current profiles through numerical computation.

In this paper, it was made a graphic evaluation of voltages and currents profiles along the half-wavelength transmission line in terms of phasor diagrams, which clarifies the reasons for this peculiar behavior.

As voltage and current vary along the line, Joule losses calculation is not a trivial task. In [7,8], is observed, by numerical computation, that Joule losses in a half-wavelength for light load conditions can be considerable. These articles proposed that voltages at both ends of the line be modulated according to the load level in order to minimize losses. In [9], which aim was to compare different transmission technologies, all scenarios considered a total voltage modulation.

As the equipment to achieve this modulation is to be designed, once it is very specific to this half-wavelength application, the cost of it should be compared to its benefit of loss reduction.

Here, it is proposed an algebraic expression for Joule power losses calculation on half-wavelength transmission system, to allow an easy and accurate calculation of Joule losses in different voltage situations.

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Expressions for long line parameters

In order to compute voltage and current phasors in any position along the line length, the used model [11], considers that series impedance and shunt capacitance are uniformly distributed along the line. Consider r as the resistance per length unit, x as the inductive reactance per length unit and b as the susceptance per length unit. The series impedance is $y = r + jx$ and the shunt admittance is $y = jb$ (shunt conductance is neglected here). The expressions for V and I, the voltage and current phasors observed in a point which distance to the load terminal is l are the solution of the following differential equations

$$\begin{aligned} \frac{d^2V}{d\ell^2} &= z\frac{dI}{d\ell} \\ \frac{d^2I}{d\ell^2} &= y\frac{dV}{d\ell} \end{aligned} \tag{1}$$

So voltage $V(\ell)$ and current $I(\ell)$ are expressed as:

$$\begin{aligned} V(\ell) &= \tfrac{V_R+I_RZ_C}{2}e^{\alpha\ell}e^{j\beta\ell} + \tfrac{V_R-I_RZ_C}{2}e^{-\alpha\ell}e^{-j\beta\ell} \\ I(\ell) &= \tfrac{I_R+V_R/Z_C}{2}e^{\alpha\ell}e^{j\beta\ell} + \tfrac{I_R-V_R/Z_C}{2}e^{-\alpha\ell}e^{-j\beta\ell} \end{aligned} \tag{2}$$

where $V_R = V(\ell = 0)$ and $I_R = I(\ell = 0)$ are voltage (phase-to-ground) and current phasors and the load terminal, Z_C is the characteristic impedance of the line and α and β are the real and imaginary parts of $\gamma = (zy)^{1/2}$.

As Z_C and γ are complex quantities, their real and complex parts are expressed as:

$$Z_C = \sqrt{\frac{z}{y}} = \sqrt{\frac{\sqrt{r^2+x^2}+x}{2b}} - j\sqrt{\frac{\sqrt{r^2+x^2}-x}{2b}} \tag{3}$$

$$\alpha = \mathrm{Re}\{\sqrt{zy}\} = \sqrt{\frac{b}{2}\cdot(\sqrt{r^2+x^2}-x)} \tag{4}$$

$$\beta = \mathrm{Im}\{\sqrt{zy}\} = \sqrt{\frac{b}{2}\cdot(\sqrt{r^2+x^2}+x)} \tag{5}$$

According to (3), the imaginary part of characteristic impedance Z_C is negligible compared to the real part because for a high-voltage transmission line, r is very small if compared to x. So Z_C can be approximated to a pure resistance. Nevertheless, the losses cannot be neglected due to the length of the line.

If an resistance equal to Z_C is connected to load terminal, the power in the load is named *surge impedance loading* (*SIL*)

$$SIL = \frac{|V_R|^2}{Z_C} \tag{6}$$

The first terms in the right-hand of (2) increase their magnitude and phase angle with x; they are named *incident* voltage and current. The terms which decrease the magnitude and angle with ℓ are named *reflected* voltage and current. Eq. (2) can be written in terms of incident and reflected phasors:

$$\begin{aligned} V_{inc,0} &= \tfrac{V_R+I_RZ_C}{2} \\ V_{ref,0} &= \tfrac{V_R-I_RZ_C}{2} \end{aligned} \tag{7}$$

$$\begin{aligned} I_{inc,0} &= \tfrac{I_R+V_R/Z_C}{2} \\ I_{ref,0} &= \tfrac{I_R-V_R/Z_C}{2} \end{aligned} \tag{8}$$

$$\begin{aligned} V(\ell) &= V_{inc,0}e^{\alpha\ell}e^{j\beta\ell} + V_{ref,0}e^{-\alpha\ell}e^{-j\beta\ell} \\ I(\ell) &= I_{inc,0}e^{\alpha\ell}e^{j\beta\ell} + I_{ref,0}e^{-\alpha\ell}e^{-j\beta\ell} \end{aligned} \tag{9}$$

Voltage and current profiles

Loading of the line will be represented by a variable impedance, as shown in Fig. 1, where the distributed impedance and admittance is represented as well as the load:

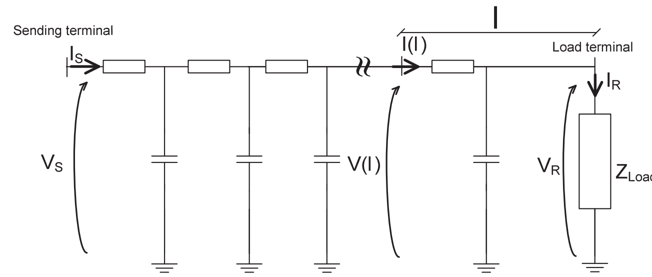

Fig. 1. Model of a long transmission line.

Voltage and current phasors will be graphically represented for each of four cases:

- Resistive load, power = *SIL*
- Resistive load, power < *SIL*
- Resistive load, power > *SIL*
- Resistive and inductive load, apparent power = |*SIL*|, power factor = 0.95.

When drawing the phasors for each of the mentioned cases, it was considered that the product $\alpha\ \ell_{total}$, where ℓ_{total} is the total length of the line, is around 5%.

Voltage profiles

Resistive load, power = SIL

For this case, power in the load is equal to *SIL* ($Z_{load} = Z_C, I_R = V_R/Z_C$), and according to (7), the reflected voltage is zero. So the resultant voltage phasors are equal to incident voltage phasors. As α is a very small value, the profile for this situation is practically flat along the line:

$$V(\ell) = V_{inc,0}e^{\alpha\ell}e^{j\beta\ell} \tag{10}$$

Resistive load, power < SIL

In order to calculate the voltage profile, the incident and reflected voltages for $\ell = 0$ (at the load terminal) are graphically evaluated in Fig. 2. In this point, reflected voltage is in-phase with incident voltage.

Voltage is graphically evaluated in four more locations in the line ($\beta\ell$ = 45°, 90°, 135° and 180°), as shown in Fig. 3.

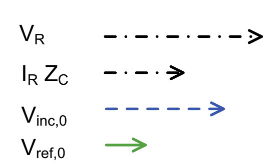

Fig. 2. Incident and reflected voltage phasors at the load, power < *SIL*.

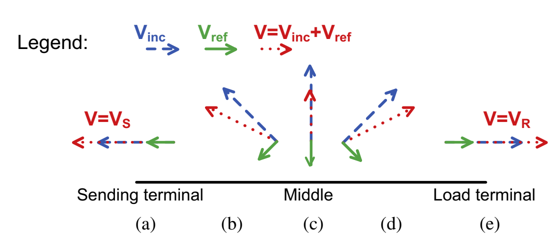

Fig. 3. Resultant voltage phasors at several locations of the line, power < *SIL*: (a) $\beta\ell = 180°$; (b) $\beta\ell = 135°$; (c) $\beta\ell = 90°$; (d) $\beta\ell = 45°$; (e) $\beta\ell = 0°$.