Electrical Power and Energy Systems 80 (2016) 179–189

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Frequency-dependent line model in the time domain for simulation of fast and impulsive transients

Pablo Torrez Caballero [a], Eduardo C. Marques Costa [b,*], Sérgio Kurokawa [a]

[a] Unesp – Univ. Estadual Paulista, Faculdade de Engenharia de Ilha Solteira – FEIS, Departamento de Engenharia Elétrica, Ilha Solteira, SP, Brazil
[b] Universidade de São Paulo – USP, Escola Politécnica, Departamento de Engenharia de Energia e Automação Elétricas – PEA, São Paulo, SP, Brazil

ARTICLE INFO

Article history:
Received 19 May 2014
Received in revised form 3 December 2015
Accepted 2 January 2016
Available online 11 February 2016

Keywords:
Bergeron method
Transmission line modeling – TLM
Electromagnetic transients
Time-domain analysis

ABSTRACT

A new transmission line model is proposed based on the well-established Bergeron method. The conventional Bergeron model is characterized by the line representation by concentrated longitudinal and transversal parameters, i.e., electrical parameters of the line are represented by means of electric circuit elements. The original approach of this research is the inclusion the frequency effect in the longitudinal parameters of the Bergeron line representation. In order to increase the frequency range covered by the proposed model, the line is represented by a cascade of line segments which are modeled following the proposed frequency-dependent Bergeron circuit. The differential equations resulted from the proposed development are represented by state matrices. The line representation by cascade of frequency-dependent Bergeron circuits enables to extend the application of the new modeling technique for simulations considering a wide range of frequencies, from a switching up to an atmospheric impulse. The proposed line model is validated based on results obtained from the well-established line model using numerical Laplace transform.

Introduction

There are several transmission line models available in the technical literature to study electromagnetic transients in power transmission systems. Basically, these models may be classified into two general groups: by lumped parameters or by distributed parameters.

In the first group, transmission lines are modeled from the representation by lumped elements, i.e., line is modeled by an equivalent representation by means of electric circuits composed of resistive, inductive and capacitive elements. These models are developed directly in the time domain, which means that can be applied for transient simulations including time-variable and non-linear elements, as: metal-oxide surge arresters, relays, non-linear loads and many other power components. This characteristic is the principal advantage in the transmission line modeling (TLM) by lumped elements [1]. The line representation by lumped parameters is well established in the technical literature for simulation of electromagnetic transients as well as other applications for power flow studies, fault location through long transmission lines and steady state phenomena [2–4].

The line modeling by distributed parameters is developed directly from the frequency-dependent parameters of the line representation by two-port circuit in the frequency domain. From this approach, the line modeling and simulations are carried out in the frequency domain and time-domain results are obtained using numeric transforms [5]. The frequency-dependent parameters of the line are accurately represented using frequency-domain models; however, these models have restrictions for inclusion of time-variable elements in the simulation process, since most power components are well established and modeled in the time domain [6].

Despite line models by lumped elements are developed in the time domain, the frequency effect on the longitudinal parameters can be included in the model using fitting methods. New frequency-dependent models based on the electric circuit approach have been described in the technical literature on TLM. These models are developed directly in the time domain from the line representation by cascade of π circuits, where the frequency effect on the electrical parameters is fitted by rational functions $R_{fit}(\omega)$ and $L_{fit}(\omega)$ (resistance and inductance) based on the longitudinal impedance of the line $Z(\omega)$, which is calculated taken into account the earth-return impedance (soil effect) and the skin effect on the cables. For example, the frequency-dependent line model described in Refs. [7,8] shows to be robust and accurate for most of transient conditions on power transmission systems.

* Corresponding author.
E-mail addresses: educosta@pea.usp.br (E.C. Marques Costa), kurokawa@dee.feis.unesp.br (S. Kurokawa).

http://dx.doi.org/10.1016/j.ijepes.2016.01.051
0142-0615/

However, depending of the transmission system characteristics (source, line and load) and transient conditions, the frequency-dependent model based on cascade of lumped elements shows to be costly in computational terms, depending of the quantity of line equivalent circuits in the cascade, total simulation time and integration step. Furthermore, hard unbalanced conditions could lead to some inaccuracies because the multi-phase modeling using a constant and real transformation matrix for the line decoupling into the respective propagation modes. Thus, the line representation by frequency-dependent cascade of π circuits shows to be efficient for several situations; however, some restrictions in the modeling procedure and inaccuracies are observed for specific cases. From this last statement, the same fitting procedure in Refs. [7,8] is also applied for the proposed line model based on the Bergeron method for simulation of fast and impulsive transients.

The Bergeron method, also known as method of the characteristics, was firstly proposed to solve hydraulic systems and after applied to electrical problems, more specifically, electromagnetic wave propagation along a lossless line [9]. In this case, the line modeling is carried out considering only the longitudinal inductance L and the shunt capacitance C, which means that the line resistance R and transversal conductance G are neglected. Thereafter, H.W. Dommel proposed a nodal solution combining the method of the characteristics for transmission lines with losses and the integration method of the trapezoidal rule for lumped parameters. The losses in the Bergeron's line model were represented by constant lumped resistances located at the sending and receiving ends of the equivalent circuit. The Bergeron's method with losses was included in the well-known *Electromagnetic Transient Program* (EMTP) [10].

A first approach for inclusion of the frequency effect in transmission line models, direct in the time domain, was described in [11]. An extension of the Bergeron's method of characteristics was developed for transmission lines with frequency-dependent parameters. However, the frequency-dependent parameters were included in the line model using inverse transforms and convolutions, which also results in several restrictions in simulations of time-variable power components and non-linear phenomena.

The proposed model is represented by a cascade of Bergeron's circuits, which results in an accurate line model capable of simulating electromagnetic transients composed of a wide range of frequencies, differently of most models developed by lumped parameters and the classical Bergeron model itself. Thus, the inclusion of the frequency effect in the Bergeron model and the line representation by cascade of lumped elements are the main contribution of this research.

This paper is structured into three parts. The first part is an introduction of the classical Bergeron model for transmission lines without losses and for lossy lines using constant parameters. The second part describes the inclusion of the frequency effect in the Bergeron model using vector fitting and the line representation by cascade of frequency-dependent Bergeron circuits. The third part validates the proposed time-domain model comparing results with a well-established model using numerical Laplace transform (NLT line model) [5]. Two signals are evaluated for the two line models: switching impulse (composed of low frequencies) and an atmospheric impulse (composed of low up to very high frequencies).

The Bergeron line model

The Bergeron's method was firstly applied for lossless transmission lines. This means that only the line inductance per unit of length (p.u.l.) L' and the p.u.l. capacitance C' were included in the model, whereas the longitudinal p.u.l. resistance R' and the p.u.l. conductance G' were neglected. In fact, the method of the characteristics could be applied for lossy transmission lines, but the resulting ordinary differential equations could not be directly integrated. Thus, considering a single-phase line with length l, the current and voltage at a point x along the line are expressed as follows:

$$-\frac{\partial e}{\partial x} = L'\frac{\partial i}{\partial t} \tag{1}$$

$$-\frac{\partial i}{\partial x} = C'\frac{\partial e}{\partial t} \tag{2}$$

The first hand of (2) and (3) represents the voltage and the current as a function of the distance x along the line, i.e., voltage and current wave propagation along the line as a function of the time t.

The general solutions of (1) and (2) are expressed [9]:

$$i(x,t) = f_1(x - \upsilon t) + f_2(x + \upsilon t) \tag{3}$$

$$e(x,t) = Z_0 f_1(x - \upsilon t) + Z_0 f_2(x + \upsilon t) \tag{4}$$

Terms f_1 and f_2 are arbitrary functions of $(x \pm \upsilon t)$. Function f_1 represents the forward wave propagation along the line with velocity υ (also known as propagation or phase velocity) whereas f_2 represents the wave propagation in a back forward direction. The line characteristic impedance Z_0 and the propagation velocity υ are expressed in the technical literature as [10]:

$$Z_0 = \sqrt{\frac{L'}{C'}}; \quad \upsilon = \frac{1}{\sqrt{L'C'}} \tag{5}$$

Multiplying (3) by the characteristic impedance Z_0 and adding in (4), the following formulation is obtained [9]:

$$e(x,t) + Z_0 i(x,t) = 2Z_0 f_1(x - \upsilon t) \tag{6}$$

$$e(x,t) - Z_0 i(x,t) = -2Z_0 f_2(x + \upsilon t) \tag{7}$$

Analyzing (6), $(e + Z_0 i)$ is constant for $(x - \upsilon t)$. The same instance is valid for the voltage $(e - Z_0 i)$ in association with $(x + vt)$. These constants are intrinsic related to the propagation characteristics and differential equations of a lossless transmission line.

Since $(x \pm \upsilon t)$ is constant, the traveling time of an electromagnetic wave from the sending end to the receiving end of the line is also constant and is expressed as:

$$\tau = \frac{l}{\upsilon} = l\sqrt{L'C'} \tag{8}$$

The equivalent circuit for a lossless line is described in Fig. 1.

The forward wave is constant from the node k to the node m at instant $t - \tau$. Analogously for a back forward wave from the node m to k. This means that the forward wave, which takes τ seconds to travel from the sending end k to the receiving end m, has the same magnitude than the back forward wave from terminal m to k, because no losses are considered in the line modeling which leads to an undefined number of wave reflections between the line terminals [9].

The historical currents for the Bergeron's line model without losses, defined in Fig. 1, are expressed as [10]:

$$I_{k,m}(t) = \frac{1}{Z_0}e_k(t) - I_k(t - \tau) \tag{9}$$

$$I_{m,k}(t) = \frac{1}{Z_0}e_m(t) - I_m(t - \tau) \tag{10}$$

Terms I_k and I_m are equivalent current sources at the sending and the receiving ends of the line, respectively. Sources I_k and I_m