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A new transfer impedance based system equivalent model for voltage stability analysis

Yang Wang [a], Caisheng Wang [a,*], Feng Lin [a], Wenyuan Li [b,c], Le Yi Wang [a], Junhui Zhao [a]

[a] Department of Electrical and Computer Engineering, Wayne State University, Detroit, MI, USA
[b] School of Electrical Engineering, Chongqing University, Chongqing, China
[c] BC Hydro and Power Authority, Vancouver, Canada

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ABSTRACT

This paper presents a new transfer impedance based system equivalent model (TISEM) for voltage stability analysis. The TISEM can be used not only to identify the weakest nodes (buses) and system voltage stability, but also to calculate the amount of real and reactive power transferred from the generator nodes to the vulnerable node causing voltage instability. As a result, a full-scale view of voltage stability of the whole system can be presented in front of system operators. This useful information can help operators take proper actions to avoid voltage collapse. The feasibility and effectiveness of the TISEM are further validated in three test systems.

Introduction

Due to increasing load demands and various pressing constraints such as economic considerations and environmental regulations, power systems are forced to operate closer to their operating limits and become more prone to voltage instability. In recent years, a considerable number of voltage instability related outage events have occurred around the world and resulted in major system failures such as the U.S.–Canada blackout on August 14, 2003 [1]. Voltage stability has become a major concern in power system planning and operation.

“Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition” [2]. Unlike angle instability, voltage instability often starts in a local network and gradually extends to the whole system. This feature makes the evolution of system losing voltage stability generally slower (in a few seconds or even longer) than that of losing angle stability which could happen quickly in a couple of cycles. Though some voltage instability phenomena can happen really fast, the focus of this paper is given to the long-term voltage stability issues.

It has been observed that voltage magnitude is not a good indication for power system voltage stability estimation [3]. In recent years, therefore, many new voltage stability indices have been proposed in literatures and some of them have been applied in real power systems [4], including the *P*–*V* and *Q*–*V* curves based methods [5,6], Jacobian matrix singularity indices [7–9], voltage collapse index based on the distance of power-flow solution pairs [10], L index [11], line-based indices [12–17] and the node-based indices [18–26].

No matter what type of indices is used in voltage stability analysis, one of critical pieces is to obtain an accurate model for the power system under study. A new system equivalent model using the concept of transfer impedance is proposed in this paper, based on which a voltage stability index named equivalent node voltage collapse index (*ENVCI*) is chosen to evaluate system voltage instability. Compared to other system equivalent methods [27–28], the proposed method has several unique characteristics: (1) generator internal impedances are included; (2) loads are substituted by corresponding equivalent impedances and included in the system impedance matrix; and (3) the impact of generators on the vulnerable nodes can be quantified and ranked by calculating the transfer power. Therefore, the TISEM can be used not only to identify the weakest nodes (buses) causing system voltage instability, but also to evaluate the impact of generators on the weakest nodes (buses). This feature is especially useful when there are distributed generators deployed in the system [29].

* Corresponding author. Address: Department of Electrical and Computer Engineering, Wayne State University, Detroit, MI 48202, USA. Tel.: +1 313 577 8074.

E-mail addresses: wangyanghh@hotmail.com (Y. Wang), cwang@wayne.edu (C. Wang), flin@ece.eng.wayne.edu (F. Lin), wen.yuan.li@bchydro.com (W. Li), lywang@wayne.edu (L.Y. Wang), Junhui.Zhao@wayne.edu (J. Zhao).

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The rest of the paper is organized as follows: 'Models and indices' presents the transfer impedance model, the *ENVCI*, and the approach of transfer power calculation. The feasibility and effectiveness of the proposed model and indices are verified in 'Simulation results' on three test systems, followed by the conclusions drawn in 'Conclusions'.

Models and indices

Transfer impedance-based system equivalent model (TISEM)

As shown in the left part of Fig. 1, a grid can be represented by an active grid with an impedance matrix Z_N and a load impedance Z_k when seen from load node k. Using the superposition theorem, the system state is the combination of two components: the component without considering the load current I_k and the additive component only reflecting the impact of the load current (see the right part in Fig. 1).

The voltage at node j in the system can be expressed as:

$$V_j = V_j^0 - Z_{jk} I_k \tag{1}$$

where V_j^0 denotes the component of node voltage without Z_k (i.e., bus k is open-circuited); I_k is the load circuit current through Z_k; Z_{jk} is the mutual impedance between nodes j and k. According to the definition of Z_{jk}, the additive voltage component at node j can be calculated as $Z_{jk}\,(-I_k)$. The negative sign here means the negative-current injection at node k. Superscript "0" is used afterwards in the paper to denote the system states or variables when load Z_k is not added.

The formula given in (1) is general and can also be used for node k:

$$V_k = V_k^0 - Z_{kk} I_k \tag{2}$$

where Z_{kk} is the self-impedance seen from the load node k.

On the other hand, for node k, we have

$$V_k - Z_k I_k = 0 \tag{3}$$

From (2) and (3), the load current can be computed as

$$I_k = \frac{V_k^0}{(Z_{kk} + Z_k)} \tag{4}$$

Given m generators in the grid, the grid is re-drawn in Fig. 2(a) by separating all the generators from the active grid.

If only one generator (e.g., the ith generator) is running in the grid and the other generators are removed (see Fig. 2(b)), using (4) and the superposition theorem, the contribution from the ith generator to the total load current (I_k) at bus k can be calculated by

$$I_{k(i)} = \frac{V_{k(i)}^0}{(Z_{kk} + Z_k)} \tag{5}$$

where $I_{k(i)}$ denotes the contribution to the load current from the ith generator; $V_{k(i)}^0$ is the voltage component at the load node k when only the ith generator is active and Z_k is not connected to the system.

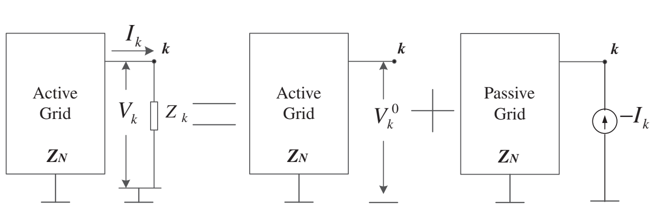
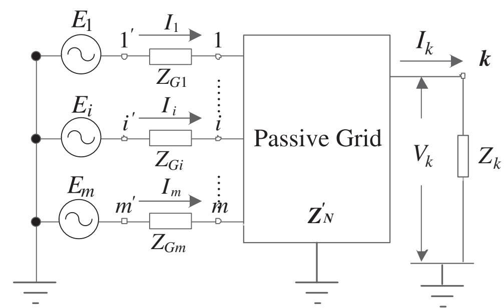
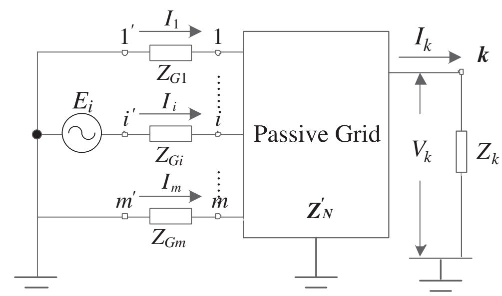


Fig. 1. Separation of a grid.



(a) Original network with generators separated



(b) Network with only one generator activated

Fig. 2. Application of superposition theorem.

Based on the definition of the mutual impedance in impedance matrix $\boldsymbol{Z_N}$, the voltage component at node k when Z_k is not connected is

$$V_{k(i)}^0 = Z_{ik} \frac{E_i}{Z_{Gi}} \tag{6}$$

where E_i and Z_{Gi} are the voltage and the internal impedance of the ith generator; Z_{ik} represents the mutual impedance between nodes i and k. $I_{i(i)}$ represents the current injection at node i when only the ith generator is active.

Substituting (6) into (5) to eliminate $V_{k(i)}^0$ yields

$$I_{k(i)} = \frac{Z_{ik}}{(Z_{kk} + Z_k)} \frac{E_i}{Z_{Gi}} = \frac{E_i}{Z_{Tik}} \tag{7}$$

where $Z_{Tik} = \frac{(Z_{kk}+Z_k)Z_{Gk}}{Z_{ik}}$, which is the transfer impedance between the ith generator (E_i) and bus k with load impedance Z_k included.

The network in Fig. 2(a) can be converted into an equivalent system using the concept of transfer impedance, as shown in Fig. 3(a). In the figure, Z'_{Tik} is the transfer impedance between the ith generator and bus k without including load impedance Z_k. Fig. 3(a) can be further converted into an overall equivalent circuit of Fig. 3(b), in which the equivalent impedance Z_{eq} and voltage E_{eq} are $\left(\sum_{i=1}^{m} \frac{E_i}{Z'_{Tik}}\right)$ and $Z_{eq}\left(\sum_{i=1}^{m} \frac{E_i}{Z'_{Tik}}\right)$, respectively. It can be readily proven that Z_{Tik} has a simple relationship with Z'_{Tik} as follows:

$$Z'_{Tik} = \alpha Z_{Tik} \tag{8}$$

where $\alpha = \frac{1}{(1+Z_k/Z_{eq})}$ and

$$Z_{eq} = \left(\sum_{i=1}^{m} \frac{1}{Z_{Tik}}\right)^{-1} - Z_k \tag{9}$$

The derivation process given in (1)–(9) are rigorously based on circuit theories, which guarantees the equivalence of the model given in Fig. 3 at the circuit level. The transfer power (discussed more in the following subsection) from generators to load nodes can be easily calculated from the TISEM, which makes the method