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A simple power factor calculation for electrical power systems

Lütfü Sarıbulut *

Adana Science and Technology University, Department of Electrical-Electronics Engineering, Adana, Turkey

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ABSTRACT

The accurately and fast estimation of Phase Difference (*PD*) is required between the voltage and current of an AC electrical power system to calculate the Power Factor (*PF*) for defining how effectively the electrical energy is converted into the useful form. Many complex methods based on difficult mathematical equations are presented by the researchers to estimate the *PD*. In this study, a new and simple algorithm derived by using the trigonometric functions is proposed for *PD* estimation to calculate *PF* of a power system. With this method, the fast-time and unaffected by distorted sinusoids of *PD* estimation are carried out by decreasing the number of mathematical equations. The performance of the proposed method is evaluated under the various system conditions by performing the simulation case studies. The results of these studies are given to verify its effectiveness under the distorted system conditions.

Introduction

The rapid developments of the power electronics technology rise the using of controlled AC drives, power converters and non-linear loads, causing the Power Quality Disturbances (PQD). These disturbances can be defined in terms of waveform distortion at the power signals and increasing of the reactive power consumption. They can cause the power loses, terminal voltage drops and negatively affects the safety, quality and economic efficiency of the electricity service. The new electrical politicizes and strategies are required for the operation and management of the electricity service in order to prevent these negative effects and to maintain the reliability and quality.

The term *Power Factor* (*PF*) has emerged with the increasing of the reactive power consumption in the electrical distribution systems recently. This term is used to express how effectively the electrical energy is converted into useful form and simultaneously indicating the quality of the service to the electricity authorized and end-users. *PF* is the ratio between the active (P) and the apparent (S) powers. It is also defined as the cosine value of the *Phase Difference* (*PD*) between the voltage (V) and current (I) of an AC electrical power system. It is given in (1).

$$PF = \cos(\varphi) = P/S \tag{1}$$

where φ is the angle value of *PD* between the two sinusoids signals.

The industrial companies try to maintain *PF* of their system close to unity. In other word, an ideal value of *PF* is 1.0, known as unity power factor. If the value of *PF* is under 0.95, it is considered to be poor. Then, its correction can be typically achieved by increasing this value to 0.95–1.0. A poor power factor is generally caused by the distorted current and inductive loads, drawing the reactive power from the utility distribution system. This situation results with the increasing of the phase difference between the voltage and current at the load terminals. Hence, the fast and accurately estimation of *PD* is required to calculate the *PF* for taking the precautions how effectively the value of *PF* can be set to the unity. It has also important significance in the engineering area, such as system model identification, intelligent control, industrial automation, and system characteristic analysis and failure diagnosis [1].

In the literature, the need for the estimating of *PD* between the power signals is justified by numerous papers for the engineering area. In these studies, several types of Zero Crossing [2,3] with their electronic circuit implementations, Discrete Fourier Transform [4–6], Sine-Wave Fit [7] and Ellipse-Fit Methods [8] are proposed. These estimation methods use some form of interpolation between points by using the correlation function to obtain a better resolution. The other types are based on Quadrature Delay Estimator (QDE) if the source is a real-valued sinusoid. These techniques utilize the in-phase and quadrature-phase components of one of the receiver outputs and provide a high-resolution phase-shift estimate [9–11]. The applications of QDE based methods are simple according to the other types of *PD* estimation methods.

The quadrature-phase component is obtained from delayed versions of the input signal such as phase-shifting as the integer number times of $\pi/2$ [9] and time-shifting as the integer number samples [10,11]. If the frequency of the signal is known, phase-shifting could be achieved as exactly $\pi/2$ by an integer number

* Tel.: +90 322 455 00 00; fax: +90 322 455 00 39.
E-mail addresses: lsaribulut@adanabtu.edu.tr, lsaribulut@hotmail.com

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of sampling periods. However, calculating QDE with a non-integer number of signal periods causes a bias in estimation [11]. The time-shifting methods are not enough for the estimation of *PD* due to that the additive methods are used for improving their performance [10,11]. Also, these methods are lack of why the quadrature-phase components are used and where the equations of *PD* estimation are derived.

This paper presents a new and simple method derived by using the trigonometric functions is proposed for *PD* estimation to calculate the *PF* of a power system. The much faster and accurate of *PD* estimation are provided by minimizing the number of mathematical equations. The theoretical explanation is given in detail by -supporting its implementation by using simulation program named as Power System Computer Aided Design (PSCAD). The proposed method is evaluated by performing simulation cases under the various system conditions. The results of case studies are given to demonstrate its effectiveness under the distorted system conditions.

The rest of the paper is organized as follows. Section 'Mathematical derivation of the proposed method' presents the mathematical derivation of the proposed method. Section 'Improvement of the proposed method for Power Quality Disturbances' describes the improvement of the proposed method for power quality disturbances. Section 'Case studies' reports the evaluation of the case studies by comparing the simulation results with their real values. Section 'Conclusions' concludes this paper.

Mathematical derivation of the proposed method

The main variables of a single-phase power system are *V* and *I*, drawn by the electrical circuit elements. If there are no harmonic source and non-linear loads in the system, these variables can be expressed by using the trigonometric functions given in the following.

$$V = V_m \cdot \sin(w_0 t + \varphi_1) \tag{2}$$

$$I = I_m \cdot \sin(w_0 t + \varphi_2) \tag{3}$$

where w_0 is the angular velocity calculated by taking the derivative of the angular displacement in a per-unit time ($w_0 = d\varphi/dt$), $V_m - I_m$ and $\varphi_1 - \varphi_2$ are the maximum values and unknown initial phases of *V* and *I*, respectively.

The calculation of main variables known as reference voltage (V_r) and current (I_r) gets complicated for three-phase -phase power system. They are also used to calculate the other variables such as *P* and reactive power (*Q*), *PF*, and phase angle. If there are no non-linear loads and harmonic source in the system, the instantaneous value of V_r and I_r can be expressed by using complex functions given in the following.

$$V_r \cdot e^{j(w_0 t+\varphi_1)} = \frac{2}{3}\left(V_a \cdot e^{jw_0 t} + V_b \cdot e^{j(w_0 t+2\pi/3)} + V_c \cdot e^{-j(w_0 t+2\pi/3)}\right) \tag{4}$$

$$I_r \cdot e^{j(w_0 t+\varphi_2)} = \frac{2}{3}\left(I_a \cdot e^{jw_0 t} + I_b \cdot e^{j(w_0 t+2\pi/3)} + I_c \cdot e^{-j(w_0 t+2\pi/3)}\right) \tag{5}$$

where $V_{a,b,c} - I_{a,b,c}$ are the phase-voltages and the line-currents of a three-phase system, respectively.

In the complex ($x - jy$) space, V_r and I_r rotate with the speed of w_0. According to the load characteristic of the power system, either

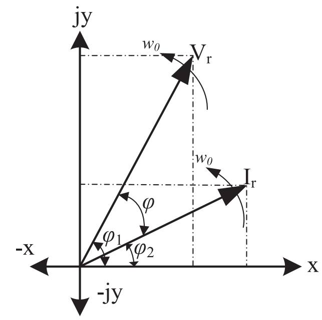

Fig. 1. Vector positions of V_r and I_r in the $x - jy$ space having the inductive characteristic.

I_r tracts V_r with the positive *PD* (φ, where $\varphi_1 > \varphi_2$) known as inductive characteristic or V_r tracts I_r with the negative *PD* ($-\varphi$, where $\varphi_1 < \varphi_2$) known as capacitive characteristic. For instance, the vsituation of the inductive characteristic is illustrated in Fig. 1.

The velocities of V_r and I_r equal to each other due to the having the same frequencies. This situation supplies the constant *PD* between these variables. Then, *PD* can be calculated by subtracting the initial phases (φ_1, φ_2) with each other for $t = 0$. It is expressed in the following form.

$$\varphi = \varphi_1 - \varphi_2, \quad \text{where } t = 0 \tag{6}$$

For $t > 0$, (6) can be written in the form of time-bounded by adding and subtracting of the term "*wt*" to the right side of the equation. Then, if a simple arrangement is made for the trigonometric functions, it can be rewritten in the following form.

$$\begin{aligned}\varphi &= wt - wt + \varphi_1 - \varphi_2, \qquad \text{where } t > 0\\ &= (wt + \varphi_1) - (wt + \varphi_2)\end{aligned} \tag{7}$$

In a three-phase power system, *P* and *Q* are calculated by using the line currents and the phase voltages in (8) and (9) without using φ in the following form [12].

$$P = V_a \cdot I_a + V_b \cdot I_b + V_c \cdot I_c \tag{8}$$

$$Q = (I_a \cdot (V_c - V_b) + I_b \cdot (V_a - V_c) + I_c \cdot (V_b - V_a))/\sqrt{3} \tag{9}$$

The derivation of φ equals to w_0 in the complex space and gives the ratio between *Q* and *P* in *PF* triangle known as "$S^2 = P^2 + Q^2$". Also, this ratio can be calculated by taking the tangent value of φ. In the proposed method, the tangent value of (7) is taken to calculate φ and written in the following form.

$$\tan(\varphi) = \tan((wt + \varphi_1) - (wt + \varphi_2)) \tag{10}$$

The right side of (10) can be separated to its sinus and cosines components and rearranged by using the sum and difference formulas of the trigonometric functions. Hence, $\tan(\varphi)$ becomes an equation expressed by using sinus and cosines functions. After the simplifications, the last form of (10) is given in the following.

$$\tan(\varphi) = \frac{\sin(wt+\varphi_1)\cdot\cos(wt+\varphi_2) - \cos(wt+\varphi_1)\cdot\sin(wt+\varphi_2)}{\cos(wt+\varphi_1)\cdot\cos(wt+\varphi_2) + \sin(wt+\varphi_1)\cdot\sin(wt+\varphi_2)} \tag{11}$$

The right side of (11) are multiplied and divided with the terms V_m and I_m and then arranged to make it similar with (2) and (3). The last statement of $\tan(\varphi)$ is written in the following form.

$$\tan(\varphi) = \frac{(V_m \cdot \sin(wt + \varphi_1))(I_m \cdot \cos(wt + \varphi_2)) - (V_m \cdot \cos(wt + \varphi_1))(I_m \cdot \sin(wt + \varphi_2))}{(V_m \cdot \cos(wt + \varphi_1))(I_m \cdot \cos(wt + \varphi_2)) + (V_m \cdot \sin(wt + \varphi_1))(I_m \cdot \sin(wt + \varphi_2))} \tag{12}$$