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A novel and fast algorithm for locating minimal cuts up to second order of undirected graphs with multiple sources and sinks

Ali Emadi, Hossein Afrakhte *

Electrical Engineering Group, Department of Engineering, University of Guilan, P.O. Box 41635 3756, Rasht, Iran

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ABSTRACT

Among various methods of evaluating reliability of a system, those based on minimal cuts (MC) are more advantageous. Calculating reliability of a system is easier by means of MCs. In addition, MCs locate unreliable parts of a system and help the engineer to improve the reliability of the system. Many algorithms have been investigated to enumerate MCs of a network. In this paper, a new and fast algorithm is presented that can deal with any undirected graph with multiple sources and sinks (e.g. power transmission and distribution systems). By defining the new concept of minimal cycles of the graph, first and second order MCs can be easily searched. Our results show that the proposed algorithm can find MCs up to second order in any undirected graph through a fast process.

Assumptions

(1) Each node is perfectly reliable.
(2) The graph is connected, undirected, free of self-loops and does not have any parallel edge.
(3) Each edge has two states: working or failed. The states of edges are statistically independent.
(4) All flows in the network obey the conservation law.

Introduction

Reliability is one of the most important indices for an engineer when planning and designing a system [1–3]. Reliability evaluation is significant while operating and controlling the system as well. Due to the importance of this subject, different techniques have been proposed to evaluate the reliability of the system. Conditional probability approach, cut set, tie set, event trees, and fault trees are some known techniques to determine the system reliability [4,5].

Among the aforementioned techniques, the cut set method is more advantageous. Systems representing a network such as communication systems, power transmission and distribution systems and transportation systems can be evaluated in terms of reliability by finding minimal cuts (MC) of the network graph. By finding MCs, the system reliability can be calculated using a simple disjoint product equation. In addition, MCs indicate unreliable parts of the network and inform the engineer to improve the system reliability [6]. A cut set is a set of system components that if all of them fail, the system fails. An MC is a cut set that all of its components must fail for the system failure and if any of members does not fail, the system will continue its work [4,7]. The order of an MC is the number of members it is consisted of.

A vast number of algorithms have been proposed to solve MC problem. Some algorithms are based on minimal paths (MP) [8]. An MP is a set of components of a system that constructs a path between source and sink node of the system, and if any of them is omitted the path will be cut. In such algorithms, it is essential to enumerate MPs of the network at first. Hence, some algorithms have been suggested for this subject [6,9–11] as well. In these methods there is the ability to search for MCs up to the desired order of engineer. However, finding all MPs of large systems is too time consuming and is impractical. Some algorithms find MCs directly. Each of them uses a particular logic to search for MCs [12–16]. The searching process for all MCs, is a time intensive process, and grows exponentially with number of network nodes.

In many systems, it is not required to deduce all of the MCs. In networks with high reliability components, if the lowest order of MCs is "n", then it is just enough to search for MCs up to "$n+1$" order. Power transmission system is one of such systems. In most power transmission systems the lowest order of MC is of the first one, if the substations are considered [5]. Therefore, it is sufficient to enumerate first and second order MCs. Moreover, in reliability

* Corresponding author. Tel.: +98 131 6690274 8; fax: +98 131 6690271.
E-mail addresses: ali_emadi@msc.guilan.ac.ir (A. Emadi), ho_afrakhte@guilan.ac.ir (H. Afrakhte).

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Nomenclatures

Acronyms

MC/MP minimal cut/path
FOMC first order MC
SOMC second order MC

Notations

|•| number of elements of •.
$G(V, E)$ a connected and undirected graph with the node set V and the edge set E. For example, Fig. 1 represents a graph with $V = \{s, u, v, m, n, t\}$ and $E = \{e_{su}, e_{sm}, e_{uv}, e_{mn}, e_{vm}, e_{vt}, e_{nt}\}$
e_{uv} $e_{uv} \in E$ is the undirected edge from node u to node v.
Path $P = \{e_{x_1x_2}, e_{x_2x_3}, \ldots, e_{x_{k-1}x_k}\}$ $(e_{x_ix_j} \in E)$ is a path from node x_1 to x_k with the length of $k - 1$, e.g. $P = \{e_{su}, e_{uv}, e_{vt}\}$ is a path from node s to t of the graph in Fig. 1
Reliability the probability that there is at least one path of working edges between sources and all sink nodes (in networks with multiple sources and sinks)
Connected graph a graph which has at least a path between each of its node pairs
Cut a subset of E that if is omitted from $G(V, E)$, it is no longer connected
MC any cut that if any of its members is omitted, it is no longer a cut
FOMC the edge set $C = \{e_{uv}\}$ is an FOMC of $G(V, E)$, if $G(V, E - \{e_{uv}\})$ is no longer connected and is consisted of two connected subgraphs
SOMC the edge set $C = \{e_{uv}, e_{mn}\}$ is a SOMC of $G(V, E)$, if $G(V, E - \{e_{uv}, e_{mn}\})$ is no longer connected and is consisted of two connected subgraphs, and none of $\{e_{uv}\}$ and $\{e_{uv}\}$ is an FOMC
MC(t) an MC isolating sink node t from all source nodes
Cycle let P be a path from node u to node v of $G(V, E)$ and $e_{uv} \in E$, then $M = P \cup e_{uv}$ is a cycle. The graph in Fig. 1 has three cycles
Minimal cycle let P be the shortest path or one of the shortest paths from node u to v of $G(V, E - \{e_{uv}\})$, and $e_{uv} \in E$, then $M = P \cup e_{uv}$ is a minimal cycle. For example, for the graph of Fig. 1, there are two paths with the shortest possible length of 3 between the nodes m and v in $G(V, E - \{e_{mv}\})$
Adjacent node node u is adjacent to node v if $e_{uv} \in E$

optimization of large systems, lots of these processes have to be executed [17,18]. Thus, it is impractical to find all MCs.

In systems containing multiple sources and sinks, MCs of all sink nodes must be enumerated. Just some of the algorithms in the literature find MCs in an efficient way for these networks [13,14]. Those algorithms are effective which search for MCs of whole node pair of sources and sinks in one procedure. The algorithm in [13] is known as one of the best algorithms to search for all MCs in such networks. It enumerates MCs for all sink nodes in an efficient way and a reasonable computation time. The disadvantage of this algorithm is inability to find MCs up to the desired order. This algorithm is used to examine validity of our proposed algorithm.

The main purpose of this paper is to introduce a new algorithm for finding MCs up to second order in a minimum possible computation time in networks containing multiple sources and sinks. The algorithm is independent of the number of sources or sinks, and it can be applied to every undirected graph.

The organization of this article is as it follows: Section 'Preliminaries' explains the basic elements and fundamental concepts of our algorithm. In Section 'Description of our algorithm' we present the proposed algorithm in detail. In Section 'Testing the proposed algorithm' the algorithm is tested on eleven benchmark graphs and some test power transmission systems. Concluding remarks are given in Section 'Conclusion'.

Preliminaries

Before defining our algorithm, some useful results of graph theory must be introduced. In undirected and connected graphs, removing an MC from the graph will divide it into two connected subgraphs. If any member of an MC is omitted from the set, it is no longer an MC.

For a graph $G(V, E)$, the number of $|E|$ cases as $C = \{e_{uv}\}$, and $|E|(|E| - 1)/2$ cases as $C = \{e_{uv}, e_{mn}\}$ must be examined to find FOMCs and SOMCs respectively. The aim of this section is to introduce theorems to limit the space of search for FOMCs and SOMCs. By providing Theorems 1 and 2 the search space will significantly become smaller. Before presenting Theorems 1 and 2 some properties, lemmas and corollaries must be defined.

Property 1. *If C is an MC, and edge $e_{ij} \in C$, then there is not any path from node i to node j in G(V, E – C).*

The corollary below is directly derived from Property 1.

Corollary 1. *Let $C \subset E$ be an edge set, and $e_{ij} \in C$. Then, C is not an MC if there is a path from node i to node j in G(V, E – C).*

Property 2 is obtained from definition of a cycle:

Property 2. *Let edge set M be a cycle of G(V, E). Then $M - \{e_{uv}\}$ is a path from node u to node v of $G(V, E - \{e_{uv}\})$.*

Theorem 1. *If M is a cycle of G(V, E), and $e_{uv} \in M$, then $\{e_{uv}\}$ is not an FOMC.*

Proof. From Property 2, there is a path from node u to node v in $G(V, E - \{e_{uv}\})$, thus from Corollary 1, $\{e_{uv}\}$ is not an MC. □

Lemma 1. *If $C = \{e_{uv}, e_{mn}\}$ is a SOMC, then there is at least one cycle containing both e_{uv} and e_{mn}, and none of them does not belong to any cycle singly.*

Proof. Since $C = \{e_{uv}, e_{mn}\}$ is a SOMC, therefore none of $\{e_{uv}\}$ and $\{e_{mn}\}$ is an MC. Hence, from Corollary 1, there is a path from node u to v in $G(V, E - \{e_{uv}\})$ call P_{uv} and a path from node m to n in $G(V, E - \{e_{mn}\})$ call P_{mn}. From Property 1, there is no path from node u to v and from node m to n in $G(V, E - \{e_{uv}, e_{mn}\})$. $G(V, E - \{e_{uv}\})$ certainly has the path P_{uv} from node u to v, but $G(V, E - \{e_{uv}, e_{mn}\})$ does not contain any path from node u to v. After removing another edge e_{mn} from the graph, P_{uv} is cut. Thus, it means $e_{mn} \in P_{uv}$ It can be similarly proved that $e_{uv} \in P_{mn}$ from definition of a cycle, $P_{mn} \cup e_{mn}$ and $P_{uv} \cup e_{uv}$ are cycles and may be identical. Both e_{uv} and e_{mn} are members of these cycles. Hence this lemma is true. □