Electrical Power and Energy Systems 78 (2016) 873–883

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Adaptive general variable neighborhood search heuristics for solving the unit commitment problem

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ARTICLE INFO

Article history:
Received 26 October 2014
Received in revised form 2 December 2015
Accepted 17 December 2015
Available online 7 January 2016

Keywords:
Power systems
Unit commitment problem
Mixed integer nonlinear problem
Variable neighborhood search

ABSTRACT

The unit commitment problem (UCP) for thermal units consists of finding an optimal electricity production plan for a given time horizon. In this paper we propose hybrid approaches which combine Variable Neighborhood Search (VNS) metaheuristic and mathematical programming to solve this NP-hard problem. Four new VNS based methods, including one with adaptive choice of neighborhood order used within deterministic exploration of neighborhoods, are proposed. A convex economic dispatch subproblem is solved by *Lambda iteration* method in each time period. Extensive computational experiments are performed on well-known test instances from the literature as well as on new large instances generated by us. It appears that the proposed heuristics successfully solve both small and large scale problems. Moreover, they outperform other well-known heuristics that can be considered as the state-of-the-art approaches.

Introduction

The unit commitment problem (UCP) consists of determining optimal production plan for a given set of power plants over a given time horizon so the total production cost is minimized, while satisfying various constraints. Every power plant individually needs to satisfy: minimum up time (minimal number of consecutive time periods during which the unit must be turned on), minimum down time (minimal number of consecutive time periods during which unit must be turned off) and production limit constraints (lower and upper production bounds). The total production of all active plants must satisfy the required demand minding that the maximal possible production cannot be less than the sum of required demand and required spinning reserves.

The unit commitment problem can be formulated as a mixed integer nonlinear problem (MINLP). Binary variables represent the ON/OFF state of every unit for each time period, while continuous variables quantify the unit production expressed in megawatts for each time period. It is easy to conclude that the number of all possible solutions grows exponentially by increasing the number of plants. The UCP is NP-hard, which means that it cannot be exactly solved in reasonable amount of time. This holds even for moderate number of units, therefore, many heuristics have been proposed in the literature to solve the UCP approximatively.

The exact method based on dynamic programming [13,14,27,35] for solving the UCP was able to tackle only problems with small number of units. Many heuristic and metaheuristic methods have been proposed up to now for the UCP such as: priority list method [1], genetic algorithms [5,22,48,53], tabu search algorithms [38], particle swarm optimization algorithms [46,56], ant colony algorithms [42], fuzzy logic [11], artificial neural networks [9,47], evolutionary programming [21], simulated annealing [43–45]. For other UCP related problems and solution approaches, we refer the reader to [39,57] and references therein.

In this paper we propose hybrid approaches that combine Variable Neighborhood Search (VNS) metaheuristic with mathematical programming. We in fact substantially extend our conference paper [54] by considering an adaptive VNS approach. Four new VNS based methods, including one with adaptive choice of neighborhood order used within deterministic exploration of neighborhoods, are proposed. Benchmark instances were used to test our hybrid methods. They have been compared with other heuristics proposed in the literature. Moreover, we suggest new set of large size instances. Computational results show that the proposed heuristic outperforms all current heuristic approaches, while improving running times for most instances. It is especially true for the largest size instances.

The rest of the paper is organized as follows. In Section 'Problem formulation' we provide mathematical formulation of the UCP, in Section 'General variable neighborhood search for solving the UCP' we describe our method. Section 'Computational results' presents comparison of our method with existing approaches, while

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http://dx.doi.org/10.1016/j.ijepes.2015.12.031
0142-0615/

Section 'Concluding remarks' concludes the paper and offers directions for future research.

Problem formulation

Economic dispatch problem

Before introducing the UCP it is necessary to define its subproblem: the Economic Dispatch Problem (EDP). Consider n thermal units (power generation units fueled by coal, oil or gas) committed to serve a load of P^D, at minimum cost. Every (thermal) unit production is bounded from below and above, this means that each unit has minimal and maximal production capacities. The objective of the EDP is to minimize the production cost while satisfying the required load and generation limit constraints for each unit. This problem can be formulated as follows:

$$\min \quad F = \sum_{i=1}^{n} F_i(P_i)$$

subject to

$$\sum_{i=1}^{n} P_i = P^D \tag{1}$$

$$P_i^{min} \leqslant P_i \leqslant P_i^{max}, \quad \text{for } i = 1, \ldots, n \tag{2}$$

where

- P_i is the production of unit i (in MW),
- $F_i(P_i)$ is the cost of production by unit i (in \$/h),
- P^D is the demand (in MW).

Fuel cost function of each unit is set as a quadratic function [58]

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2, \tag{3}$$

where $a_i, b_i, c_i, \quad i = 1, \ldots, n$ are given coefficients.

Lambda iteration method for solving economic dispatch problem

The lambda-iteration method [58] is, so far, the most popular method for solving the EDP, when the objective function F is quadratic. It is used to iteratively determine optimal Lagrange multiplier λ which corresponds to constraint (1). The lambda iteration procedure stops when the tolerance, which indicates that the sum of all online units output minus the load demand, is less than the value given beforehand. When the Lagrange multiplier λ is known, it is simple to calculate the production of each unit by solving system of linear equations. The scheme of lambda iteration method is presented below (Algorithm 1).

Algorithm 1. Lambda Iteration method

```
Function LIM();
  λ^min ← min_{i=1,...,n} dF_i(P_i^min)/dP_i;
  λ^max ← max_{i=1,...,n} dF_i(P_i^max)/dP_i;
  ϵ ← 10^-6;
  repeat
    λ ← (λ^min + λ^max)/2;
    Calculate P_i from dF_i(P_i)/dP_i = λ;
    Δ = P^D − Σ_{i=1}^n P_i;
    if P^D > Σ_{i=1}^n P_i then λ^min = λ;
    if P^D < Σ_{i=1}^n P_i then λ^max = λ;
  until |Δ| ⩽ ϵ;
  return P_1, ..., P_n;
```

Unit commitment problem

The basic goal of the UCP is to properly schedule the ON/OFF states of all units in the system with minimum (fossil) fuel cost. The ON/OFF state of the entire system is represented by the binary matrix $U_{i,t} \in \{0,1\}$, for $i \in N = \{1, \ldots, n\}$ and $t \in H = \{1, \ldots, T\}$. In addition to fulfill a large number of constraints, the optimal UC should meet the predicted load demand requirement (7) with spinning reserves (6) at every time interval such that the total operating cost is minimal (4). Therefore, the solution of the unit commitment problem relies on iteratively solving the economic dispatch problem for all time intervals t (e.g., an hour) in overall time T respecting feasibility of the time constraints (8).

The model can be stated as follows:

$$\min \quad G = \sum_{i \in N} \sum_{t \in H} [[F_i(P_{i,t}) + ST_i(1 - U_{i,t-1})]U_{i,t} + SD_i U_{i,t-1}(1 - U_{i,t})] \tag{4}$$

subject to:

$$P_i^{min} U_{i,t} \leqslant P_{i,t} \leqslant P_i^{max} U_{i,t}, \quad i \in N, \ t \in H \tag{5}$$

$$\sum_{i \in N} P_i^{max} U_{i,t} \geqslant P_t^D + P_t^R, \quad t \in H \tag{6}$$

$$\sum_{i \in N} P_{it} = P_t^D, \quad t \in H \tag{7}$$

$$\begin{aligned} &U_{i,t} - U_{i,t-1} \leqslant U_{i,t+j} \quad i \in N; \ t \in H; \ j = 1, \ldots, T_{i,up} - 1; \\ &U_{i,t+j} \leqslant U_{i,t} - U_{i,t-1} + 1 \quad i \in N; \ t \in H; \ j = 1, \ldots, T_{i,down} - 1; \end{aligned} \tag{8}$$

$$U_{i,t} \in \{0,1\}, \quad P_{i,t} \geqslant 0 \quad i \in N; \ t \in H \tag{9}$$

where

$$ST_i = \begin{cases} HSC_i, & \text{if } T_{i,down} \leqslant T_{i,off}^t \leqslant T_{i,cold} + T_{i,down} \\ CSC_i, & \text{otherwise} \end{cases} \tag{10}$$

It is assumed that the shut down cost SD_i for every unit i is equal to zero ($SD_i = 0$).

The startup cost (ST_i) depends on how long unit i is off line. We differ two types of startup cost: cold start (CSC_i) and hot start (HSC_i). In practice, the cold start is much more expensive than a hot start.

The constraints (8) represent the minimum up and down time requirements of each unit. This means that each unit must be on line (up time) and off line (down time) for a certain consecutive time period ($T_{i,up}$, $T_{i,down}$, respectively). These two constraints represent the time constraints, while (5)–(7) are production constraints. This means that a UCP solution must fulfill both production feasibility (produce required load, bounded by system capacities, satisfying spinning reserve constraints) and time feasibility (units must be online/offline for a consecutive time period).

The presented formulation of the UCP has $nT + 2T + nT\sum_{i=1}^{n}(T_{i,up} + T_{i,down} - 2)$ constraints and $O(nT)$ binary and continuous variables.

General variable neighborhood search for solving the UCP

As we mentioned earlier, finding an optimal solution for large size the UCP is unlikely to be possible in reasonable time, and thus heuristic methods are a preferable option for finding good or near-optimal solutions. For that purpose, we propose an efficient Variable Neighborhood Search (VNS) based heuristics [16,31].

VNS is a flexible framework for building heuristics to approximately solve combinatorial and non-linear continuous optimization problems. VNS changes systematically the neighborhood structures during the search for an optimal (or near-optimal)