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Linear programming techniques for developing an optimal electrical system including high-voltage direct-current transmission and storage

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ABSTRACT

The planning and design of an electric power system, including high-voltage direct-current transmission, is a complex optimization problem. The optimization must integrate and model the engineering requirements and limitations of the generation, while simultaneously balancing the system electric load at all times. The problem is made more difficult with the introduction of variable generators, such as wind and solar photovoltaics. In the present paper, we introduce two comprehensive linear programming techniques to solve these problems. Linear programming is intentionally chosen to keep the problems tractable in terms of time and computational resources. The first is an optimization that minimizes the deviation from the electric load requirements. The procedure includes variable generators, conventional generators, transmission, and storage, along with their most salient engineering requirements. In addition, the optimization includes some basic electric power system requirements. The second optimization is one that minimizes the overall system costs per annum while taking into consideration all the aspects of the first optimization. We discuss the benefits and disadvantages of the proposed approaches. We show that the cost optimization, although computationally more expensive, is superior in terms of optimizing a real-world electric power system. The present paper shows that linear programming techniques can represent an electrical power system from a high-level without undue complication brought on by moving to mixed integer or nonlinear programming. In addition, the optimizations can be implemented in the future in planning tools.

1. Introduction

An electric power system is a complex web of power generators, transmission and distribution lines, a small amount of storage, and power consumers, which must be kept in dynamic equilibrium. The electric power generated on the system at any one instance must be consumed somewhere at the same instant. The historical design of electric power systems is an ad hoc method of addition as needed. A description of electric power systems can be found in, e.g., [1,2]. The ad hoc nature of electric power system growth and regeneration can lead to system weaknesses, which can hamper further growth of new generation and transmission. The electric power system design is an ideal candidate for mathematical optimization. There already exists research into different optimization schemes for different aspects of the power system. The research ongoing has wide ranging interests from the optimization of asset scheduling to power flow optimization across a network, for a selection of related optimizations and overviews see, e.g., [3–6].

The optimization of electric power systems becomes even more difficult with the addition of renewable generators, such as wind turbines and solar photovoltaic (PV) cells. The optimization must take into consideration the variable nature of these relatively new forms of power. In recent years, the optimization of wind, solar, and conventional generator systems has attracted strong research. Much of the attention in the research has been to consider high penetration levels of wind and solar PV deployment in the electrical power system, see e.g., [7–11], which is what we consider in the present paper. The variable nature of wind and solar resources implies that for an optimization to be an effective planning tool for electric power systems it must consider large geographic areas, with high-temporal and -spatial resolution discretization.

Since numerous objectives exist in the mathematical *optimization* of an electric power system with wind and solar PV electrical generation plants, one has to choose what is meant by an *optimal*

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system. For example, in [7,8] an energy balance optimization was performed whereby the amount of energy produced in a given time is balanced by the energy consumed in the same time interval. Unfortunately from an electric power system perspective, the optimal system that can generate enough power, but not at the correct times is not effective in the real world. The optimization of [7,8] describes only that wind and solar power can help in an electrical power system, but not to what extent. The procedure carried out by [9] is based upon the Ordinary Least Squares (OLS) and Regularized Least Squares (RLS) procedures, see e.g. [12–16], and it finds an optimal solution with respects to matching the electric load at every time step throughout the time interval studied.

The load-matching procedure set out in [9] is novel, but incomplete and incorrect in certain areas, and thus we set out a new formulation in the present paper. The optimization in [9] has the real-world drawback of possibly being extremely inefficient in terms of cost. The methods adopted in [10,11] are both defined as cost-minimization. The former developed a procedure to look at policy implications with growing the variable resource under numerous constraints. The latter performs an iterative approach to the cost-minimization and attempts to search the solution space via discretization. The cost-minimization technique is the most appropriate for real-life decision-making. In the present paper, we develop a unique cost optimization procedure that designs a wind, solar PV (or any other variable generator) and conventional electricity power generation system, while simultaneously designing a HVDC transmission system and deploying storage capabilities.

The purpose of the present paper is to derive two mathematical optimizations that consider the electric power system as a whole, which can be applied to any desired system. The variable and conventional generators, the transmission, the storage, and the electrical demand all need to be modeled in the optimization to create the most realistic system. In reviewing the literature, we did not find a single optimization procedure that performed the modeling of the electric power system holistically. The present paper describes a load-matching procedure and a cost-minimizing procedure. Both optimizations include all of the fundamental characteristics of an electric power system, however, by necessity, they cannot include every single, small, technical detail of a complete system. We develop *both* the load-matching and cost optimization procedures because it is informative to know to what extent wind and solar PV power can contribute to an electric power system, with and without the constraints of cost. We test the codes on a sample system to demonstrate its capabilities. The mathematical optimizations are designed to be used on large geographic-scale electric power systems at a high-temporal and -spatial resolution. The procedures were intentionally designed to be linear programming and not to be nonlinear programming. Choosing LP rather than NLP was done by transforming the problems while retaining all the salient features. In addition, the choice allows for large problems to be solved in a tractable amount of time and computation resources.

The layout of the present paper is as follows: Section 'Electrical load-matching technique' develops a load-matching mathematical linear programming technique for fast and efficient design of large-scale electric power systems, the section includes descriptions of each modeled parameter and its importance. Section 'Cost minimization technique' describes a mathematical cost optimization for large-scale electric power systems and discusses the important features of the procedure. In Section 'Example test case', we show a sample execution of the two mathematical optimizations on a relatively small data set. Finally, in Section 'Discussion and conclusions', we discuss the benefits and disadvantages of the procedures and the most efficient methods to use when deploying the models using GAMS/Cplex software [17].

2. Electrical load-matching technique

The technique of load-matching is to find, from a geometric perspective, the shortest total distance between all of the electricity generation and the electricity demand at each time instance over a specified time interval (usually a day, week, month, or year). To cast this problem effectively in a Linear Programming (LP) framework, we define the objective function as the sum of excess generation, backup generation, electrical losses due to transmission to consumers, and losses from moving electricity between the grid and storage. We then bound the objective function with linear constraints to ensure that the electric demand is met at every time step, the storage is charged and discharged at an appropriate rate, the transmission is constructed as necessary, the backup generation increases and decreases output (ramps) within technical bounds and the (variable) generation plants are not overbuilt.

The minimization problem modeled in the present paper is designed for wind- and solar- dominated systems, however, the methodology can be used, in principle, for optimizing any electricity generation system. It is already well known that all LP problems can be written in the standard (slack) form [18]

$$\text{minimize } f(\mathbf{x}) \triangleq \mathbf{c}^{\mathrm{T}}\mathbf{x}, \tag{1}$$

$$\text{subject to } \mathbf{Ax} = \mathbf{b}, \quad \mathbf{x} \geqslant \mathbf{0}, \tag{2}$$

where $\{c_j\} \in \mathbb{R}^n, \{x_j\} \in \mathbb{R}^n, \{A_{ij}\} \in \mathbb{R}^{m\times n}$, and $\{b_i\} \in \mathbb{R}^m$. For the sake of brevity, we drop the {} notation for the remainder of the present paper. The coefficients in Eq. (1) are known as the costs or weighting factors for each of the primary endogenous variables. The coefficients in Eq. (2) are usually known, or should be calculable from other constraints. For computational efficiency the LP methods set out in the present paper are written in the most appropriate form for compilation by the solvers. We have shown the standard form for reference to check against to make sure the model created does not become infeasible due to conflicting constraints.

The load-matching optimization developed in the present paper can be written as

$$\text{Min } \chi = \sum_{\tau}\left(\sum_{\mu} g_{\mu\tau} + c_{\mu\tau} + \mathcal{L}^{ts}_{\mu}\cdot s^{o}_{\mu\tau} - \mathcal{L}^{fs}_{\mu}\cdot s^{i}_{\mu\tau} + \frac{1}{2}\sum_{\alpha}\sum_{\beta}\mathcal{L}^{tr}_{\alpha\beta}\cdot\mathcal{D}_{\alpha\beta}\cdot\mathcal{T}_{\alpha\beta\tau}\right) \tag{3}$$

subject to:

$$\sum_{\phi}\left(b_{\phi\mu}\cdot\sum_{\kappa}x_{\phi\kappa}\cdot r_{\phi\kappa\tau}\right) + g_{\mu\tau} + t_{\mu\tau} - c_{\mu\tau} + s^{i}_{\mu\tau}\cdot\left(1-\mathcal{L}^{fs}_{\mu}\right) - s^{o}_{\mu\tau}\cdot\left(1+\mathcal{L}^{ts}_{\mu}\right) = L_{\mu\tau}, \quad \forall\mu,\tau; \tag{4}$$

$$t_{\mu\tau} = \sum_{\alpha}\mathcal{T}_{\alpha\beta\tau}\cdot\left(1-\mathcal{L}^{tr}_{\alpha\beta}\cdot\mathcal{D}_{\alpha\beta}\right)\Big|_{\beta=\mu} - \sum_{\beta}\mathcal{T}_{\alpha\beta\tau}\Big|_{\alpha=\mu}, \quad \forall\mu,\tau(\alpha\neq\beta); \tag{5}$$

$$T_{\hat{\alpha}\hat{\beta}} \geqslant \mathcal{T}_{\alpha\beta\tau}\big|_{\alpha,\beta=\hat{\alpha},\hat{\beta}} \geqslant 0, \quad \forall\hat{\alpha},\hat{\beta},\tau(\hat{\alpha}>\hat{\beta}); \tag{6}$$

$$\hat{S}_{\mu\tau} = \left[s^{o}_{\mu\tau} - s^{i}_{\mu\tau}\right] + \hat{S}_{\mu(\tau-1)}\cdot\left(1-\mathcal{L}^{rs}_{\mu}\right), \quad \forall\mu,\tau\geq 1; \tag{7}$$

$$C^{s}_{\mu} \geqslant \left(1+\mathcal{R}^{s}_{\mu}\right)\cdot s^{o}_{\mu\tau} \geqslant 0, \quad \forall\mu,\tau; \tag{8}$$

$$0 \leqslant s^{o}_{\mu\tau} \leqslant \mathcal{S}^{D}_{\mu}\cdot C^{s}_{\mu}, \quad \forall\mu,\tau; \tag{9}$$

$$0 \leqslant s^{i}_{\mu\tau} \leqslant \mathcal{S}^{C}_{\mu}\cdot C^{s}_{\mu}, \quad \forall\mu,\tau; \tag{10}$$