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Projected mixed integer programming formulations for unit commitment problem

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ABSTRACT

The thermal unit commitment (UC) problem is a large-scale mixed integer quadratic programming (MIQP), which is difficult to solve efficiently, especially for large-scale instances. This paper presents a projected reformulation for UC problem. After projecting the power output of unit onto [0,1], a novel MIQP reformulation, denoted as P-MIQP, can be formed. The obtained P-MIQP is tighter than traditional MIQP formulation of UC problem. And the reduced problem of P-MIQP, which is eventually solved by solvers such as CPLEX, is compacter than that of traditional MIQP. In addition, two mixed integer linear programming (MILP) formulations can be obtained from traditional MIQP and our P-MIQP of UC by replacing the quadratic terms in the objective functions with a sequence of piece-wise perspective-cuts. Projected MILP is also tighter and compacter than the traditional MILP due to the same reason of MIQP. The simulation results for realistic instances that range in size from 10 to 200 units over a scheduling period of 24 h show that the projected reformulation yields tight and compact mixed integer programming UC formulations, which are competitive with currently traditional ones.

Introduction

The unit commitment (UC) problem is an important problem in the power industry. The objective of the UC problem is to minimize the total operation cost of the generating units over the scheduled time horizon while satisfying the demand and reserve requirements and all the other constraints of the generating units. In general, the UC problem is formulated as a mixed integer nonlinear programming problem [1] which is nonconvex, and the scale of this problem make large UC problems challenging to solve.

In this paper, we present a projected reformulation for UC problem. By projecting the power output of unit onto [0,1], the mixed integer quadratic programming (MIQP) and mixed integer linear programming (MILP) formulations of UC problem can be transformed to projected mixed integer programming (MIP) formulations respectively. The obtained projected MIP formulations are tighter than traditional MIP formulations of UC problem. The solution logs of MIP solvers show that the reduced problems of projected MIPs, obtained by the Presolve process of MIP solvers, often have slightly fewer rows and nonzero elements than the reduced problems of traditional MIPs. This means that the projected MIPs are often compacter than the traditional ones in the sense of reduced problems. Finally, simulation results for 42 instances that range in size from 10 to 200 units for 24 h intervals show that the projected MIP formulations are very promising for large-scale UC problems.

The remaining parts of this paper are organized as follows. Section ‘Literature review’ introduces some related works in solving UC problems, especially for MIP method. In Section ‘MIP formulation for UC problem’, the traditional MIP mathematical formulation of the UC problem is introduced. With the given formulation, we present our projected MIP formulations in Section ‘Tight and compact MIP formulations’, which are formed by projecting $P_{i,t}$ onto [0,1]. Meanwhile, we explain the tightness and compactness of projected MIP formulations in this section as well. The computational results are reported in Section ‘Numerical results and analysis’ to verify the effectiveness of the proposed projected formulations. Finally, we conclude the paper in Section ‘Conclusion’.

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Nomenclature

Indices

i	index for unit
t	index for time period

Constants

N	total number of units
T	total number of time periods
α_i, β_i, γ_i	coefficients of the quadratic production cost function of unit i
$C_{\text{hot},i}$	hot startup cost of unit i
$C_{\text{cold},i}$	cold startup cost of unit i
$\underline{T}_{\text{on},i}$	minimum up time of unit i
$\underline{T}_{\text{off},i}$	minimum down time of unit i
$\underline{T}_{\text{cold},i}$	cold startup time of unit i
$\bar{P}_i$	maximum power output of unit i
$\underline{P}_i$	minimum power output of unit i
$P_{\text{D},t}$	system load demand in period t
R_t	spinning reserve requirement in period t
$P_{\text{up},i}$	ramp up limit of unit i
$P_{\text{down},i}$	ramp down limit of unit i
$P_{\text{start},i}$	startup ramp limit of unit i
$P_{\text{shut},i}$	shutdown ramp limit of unit i
$u_{i,0}$	initial commitment state of unit i (1 if it is online, 0 otherwise)
$T_{i,0}$	number of periods unit has been online (+) or offline (−) prior to the first period of the time span (end of period 0)

Variables

$u_{i,t}$	schedule of unit i in period t, binary variable that is equal to 1 if unit i is online in period t and 0 otherwise
$s_{i,t}$	startup status of unit i in period t, which takes the value of 1 if the unit starts up in hour t and 0 otherwise
$P_{i,t}$	power output of unit i in period t
$\tilde{P}_{i,t}$	projected variable of $P_{i,t}$ which belongs to [0,1], describing the location of in $[\bar{P}_i, \underline{P}_i]$
$S_{i,t}$	startup cost of unit i in period t

Literature review

Many artificial intelligence (AI) and numerical optimization methods, most of which are approximations, have been proposed to solve UC problem. Artificial intelligence (AI) methods include evolutionary algorithm [2], genetic algorithm [3], particle swarm optimization [4], artificial neural network [5], simulated annealing [6], and hybrid methods which combine two or more of the methods above [7–10]. Numerical optimization methods include priority list (PL) [11], branch and bound (B&B) [12], Benders decomposition (BD) [13], outer approximation (OA) [14,15], dynamic programming (DP) [16], Lagrangian relaxation (LR) [17], semidefinite programming (SDP) [18], relaxation method [19]. AI methods can provide fair solutions within reasonable computation time. However, the quality of the solutions is difficult to guarantee [1]. As for numerical optimization methods, most of them require impractical computation time for large-scale systems when high-quality solutions are needed.

With the significant progresses in the theory of mathematical programming and improvements in the efficiency of general-purpose MIP solvers [20–25], solving UC problem by using MIP method is becoming increasingly popular. Historically, LR has been the method of choice for UC scheduling software used by the power industry. But, the world's largest competitive wholesale market, PJM, has recently replaced LR with MIP to solve the UC-based scheduling problems [26]. General-purpose MIP solvers can provide solutions with comparable quality in comparable time. Solvers are easier to use, and the models are easier to modify to take into account other new factors. Furthermore, solvers do not only provide feasible solutions, but also optimality guarantees that prove that the solutions are indeed accurate to the required precision.

UC problem can be simply formulated as an MIQP and be solved by solvers directly [27]. Taking into account the perspective function [28,29], a well-known tool in convex analysis, UC problem can be reformulated as a tight mixed integer second order cone programming (MISOCP). However, [25] proves that directly passing the MISOCP formulation to the solver is less competitive than employing piecewise-linear approximations. After approximating the problem to an MILP formulation, UC problem can be solved by using MILP solvers. MILP method can often obtain better solutions than mixed integer nonlinear methods because, nowadays, MILP heuristics are more developed than the nonlinear ones [27]. After upper of lower approximating the quadratic objective function to be piecewise linear function [30], UC problem can be solved by using MILP method [22]. One way to achieve a very small MILP relative optimality gap within a given timeframe is to have a tight relaxation that approximates the problem better, and this will result in better lower bounds and higher efficiency in obtaining optimal integral solutions via a branch-and-cut algorithm [31]. Ref. [21] has recently proposed an MILP best approximated with gradient-based perspective cuts. Ref. [29] proposes an MILP approximating the second-order cone constraint with a linear programming whose size grows logarithmically with increasing levels of accuracy. In [23], a sequence of valid inequalities taking three types of binary variables into account is given to describe a tighter feasible region of the UC problem. Moving forward, tight and compact MILP UC formulations are provided in [24] incorporating the startup and shutdown power trajectories of thermal units.

Despite the significant improvements in MIP solving method, the time required to solve UC problems continues to be a critical limitation that restricts the size and scope of UC models. For efficiently solving large-scale UC problems, MIP formulations of UC problems should be further improved.

MIP formulation for UC problem

Objective function and constraints

The objective function of the UC problem is to minimize the total operation cost F_C. It has the form

$$\min F_C = \sum_{i=1}^{N}\sum_{t=1}^{T}[u_{i,t}f_i(P_{i,t}) + S_{i,t}] \tag{1}$$

where the production cost $f_i(P_{i,t}) = \alpha_i + \beta_i P_{i,t} + \gamma_i (P_{i,t})^2$, and $S_{i,t}$ is the startup cost.

The constraints of the UC problem are:

(1) Unit generation limits:

$$u_{i,t}\underline{P}_i \leqslant P_{i,t} \leqslant u_{i,t}\bar{P}_i. \tag{2}$$

(2) Power balance constraints:

$$\sum_{i=1}^{N} P_{i,t} - P_{\text{D},t} = 0. \tag{3}$$