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Interior–exterior point method with global convergence strategy for solving the reactive optimal power flow problem

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ABSTRACT

This paper proposes a predictor–corrector primal–dual modified log-barrier interior–exterior point method with global convergence and cubic fitting strategies for solving the Reactive Optimal Power Flow (ROPF) problem. The interior–exterior approach is a variant of the primal–dual nonlinear rescaling method, recently proposed. The application of the global convergence strategy produces only descent directions, even if the optimization problem is non-linear and non-convex. The application of a cubic fitting strategy for modified log-barrier functions preserve the continuity and also the first and second-order derivatives of the logarithm near the boundary of the feasible set. Some updating rules for the Lagrange multiplier estimates are theoretically and numerically evaluated. Numerical tests and comparisons with classical interior point methods, involving the electrical systems of 3, 9, IEEE-14, IEEE-30, IEEE-57, IEEE-118, IEEE-162 and IEEE-300 buses, are performed, which demonstrate the robustness and efficiency of the method.

Introduction

The Optimal Power Flow (OPF) is a nonlinear, non-convex and large scale optimization problem involving both continuous and discrete variables [34]. The purpose of the OPF is to determine the best operational point for the electric system through the optimization of an objective function representing a certain system performance, while satisfying operational constraints. Depending on the system performance and on the constraints adopted, many sub-classes of OPF problems may be defined. Historically, the basic subclasses described in [18] involve active and reactive optimal power flow problems. The active OPF (AOPF) is concerned with the calculation of active power controls, such as active generation and angles for phase shifter transformers, by means of the minimization of the generation costs. Examples of AOPF formulation involve some sub problems, such as: economic dispatch [31,5,7], predispatch [32], and unit commitment [36]. The reactive OPF (ROPF) is concerned with the calculation of reactive power controls, such as voltage magnitudes in controllable buses, transforms tap ratios, and capacitor/reactor banks, by means of the minimization of some reactive criteria, such as: transmissions system losses and voltage profile deviation. Finally, the active-reactive OPF (AROPF) approaches attempt to optimize both active and reactive powers jointly.

As the solution techniques used to solve the OPF problem evolved, new OPF formulations were also proposed that incorporate more realistic aspects of the problem. In such a context, we may point out the security-constrained OPF (SCOPF) and the probabilistic OPF (POPF). The SCOPF is an extension to the OPF problem, which takes into account constraints arising from the operation of the system under a set of postulated contingencies [34,14]. The POPF is used whenever uncertainties in some of the parameters of the problem, such as loads and generation, must be represented in the OPF formulation [46,3,16].

The algorithms adopted for solving these more realistic OPF formulations generally necessitate solving a huge amount of OPF sub-problems. Hence, these sub-problems demand robust local search procedures with good computational properties, such as: good matrix conditioning, small number of cycles, and good accuracy. In this paper, we compare two primal–dual predictor-corrector algorithms having such characteristics for solving the ROPF problem, namely: the interior-point method described in [44] and the modified log-barrier interior–exterior point method here proposed.

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The first attempt for solving the general ROPF model was proposed by [15], who solved it by using KKT conditions and a Gauss–Seidel method. Since then, numerous researchers have proposed a variety of methods for its solution. These researchers include [18], who used the reduced-gradient technique associated with a Newton–Raphson method; [38], using the penalty function method; [41], using a decoupled Newton–Raphson method; [17], who used the augmented Lagrangian function;[22,44], using the interior-point method (IPM); [1], by applying the modified barrier-augmented Lagrangian function; [6], using an approach of the log-barrier augmented Lagrangian function; [42], using methods involving augmented Lagrangian function and interior-point trust region; [40], using the modified barrier Lagrangian function; [45], who used a decomposition–coordination interior-point method. Recently, [39] solved the ROPF including discrete variables using filtering methods and a penalty function. Some convexity properties of the dual problem associated with the OPF were discussed in [25]. The authors also provide necessary and sufficient condition to guarantee the existence of zero-duality gap for the equivalent form of the OPF problem. These results are conceptually important since the authors have shown that the standard IEEE benchmark systems with 14, 30, 57, 118, and 300 buses satisfy these necessary and sufficient conditions. In other words, these practical systems can all be convexified via the new formulation proposed in [25].

Recent literature regarding the ROPF solution technique also includes numerical experiences with evolution algorithms (EA) such as: the evolutionary algorithm proposed in [37], that uses the concept of incremental power flow model based on sensitivities, which reduces substantially the number of power flow evaluations, resulting in solution speed up; the two stage initialization algorithm described in [4] that does not use the mutation operation and calculates the solution with less number of generations; the nondominated sorting multi objective gravitational search algorithm described in [11] for solving different ROPF problems; the Evolutionary Particle Swarm Optimization approach described in [16], that investigates the effects of wind generation on power system operation and planning by means of an OPF model. Although the experiences with EA for solving the ROPF problem have been successfully described in the literature, the results presented are generally restricted to small or medium size power systems. In contrast, as discussed in [34], the algorithms developed using the concepts of IPM are capable of solving large scale ROPF problems. However, in recent studies with IPM described in [13] the authors underline the need to improve the reliability of IPM codes for solving very difficult OPF problems. Another important drawback of EA approaches is their inability to verify the optimality of the solutions obtained. Hence, one never knows whether an optimal solution has really been reached.

This paper proposes a predictor–corrector primal–dual Modified Log-Barrier Interior–Exterior Point Method with Global Convergence and Cubic Fitting strategies (MLBIEPMGC–CF) to solve the ROPF. The modified barrier function proposed by [35] can be considered an interior augmented Lagrangian function. Contrary to the classical log-barrier function introduced by [20], the modified barrier function allows for the existence of first and second order derivatives at boundary points of the feasible set, due to its finite relaxation property in this region. The global convergence strategy adopted by the MLBIEPMGC–CF involves the use of rank correction [33] on the Hessian matrix [9] associated with the modified log-barrier Lagrangian function, using a variant of the Levenberg–Marquardt method. The purpose of this correction is to make the matrix positive definite in all the cycles of the method, which ensures that the search for local minima or the global minimum is successful within the feasible set, thereby preventing the poor conditioning of the Hessian matrix, which has been observed in the literature [40], particularly for points at the boundary of the feasible set.

In the MLBIEPMGC–CF, we adopt a cubic polynomial fitted to the modified barrier function at a predetermined point in the relaxed region. This polynomial allows the modified log-barrier Lagrangian function to be defined for points beyond and close to the boundary of this finite region. The adjustment is made so that the values of the polynomial function and its first and second order derivatives coincide, respectively, with the values associated to the modified barrier function. The advantages of fitting a cubic polynomial rather than using a quadratic polynomial, as shown by [29], are that both the curvature and the strict convexity of the modified barrier function are preserved at a predetermined point.

In what concerns the potential for solving ROPF problems, our method has the following contributions: (i) the KKT conditions are fully satisfied with great precision, and zero duality gap; (ii) the global convergence strategy assures primal steepest descent directions in all iteration which, in turn, assures that minimum rather that maximum optimal solutions are always found; (iii) ill-conditioning problems associated with the Hessian matrix are also substantially minimized by means of the global convergence strategy; and (iv) the strategy for updating the Lagrange multiplier estimates reduces the number of iterations for convergence, as shown in the results.

This paper is structured as follows. Section ‘The reactive optimal power flow problem’ describes the ROPF problem; section ‘Interior and exterior point methods’ briefly describes the main concepts related to interior and exterior point methods, in order to put the method proposed into perspective. Section ‘The primal–dual modified log-barrier interior–exterior point method’ presents the proposed predictor and corrector procedures in the context of the predictor–corrector primal–dual Modified Log-Barrier Interior Exterior Point Method (MLBIEPM); section ‘The global convergence strategy’ describes the global convergence strategy; section ‘Cubic fitting strategy’ discusses the proposed cubic fitting strategy; section ‘Updating of the barrier parameter’ describes the update of the barrier parameter; section ‘Algorithm of the MLBIEPMGC–CF method’ presents the algorithm of the MLBIEPMGC–CF method; section ‘Numerical results’ describes the numerical results, and lastly, section ‘Conclusions’ outlines our main conclusions.

The reactive optimal power flow problem

The purpose of the ROPF problem is to calculate reactive power controls of the system that minimize an optimization criterion, such as transmission losses and optimization of the voltage profile, among others, taking into account the main physical and operational constraints of the transmission system. In this work, we chose to minimize transmission losses. Mathematically, the problem is formulated as shown in (1):

$$\begin{cases} Min \quad f(\mathbf{x}) \\ s.t. \\ \qquad \mathbf{g}(\mathbf{x}) = \mathbf{0} \\ \qquad \mathbf{u_1} \leqslant \mathbf{h}(\mathbf{x}) \leqslant \mathbf{u_2} \\ \qquad \mathbf{l_1} \leqslant \mathbf{x} \leqslant \mathbf{l_2}, \end{cases} \tag{1}$$

where

- $\mathbf{x} = [\mathbf{V}, \gamma]$: is the dimensional vector of voltage magnitude and angles, respectively;
- $\mathbf{l_1}$ and $\mathbf{l}_2$: are the vectors of the minimum and maximum limits of $\mathbf{x}$, respectively;
- $f(\mathbf{x})$: is the function that represents the sum of active power losses in transmission;