Electrical Power and Energy Systems 35 (2012) 171–179

Contents lists available at SciVerse ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Algebraic-graph method for identification of islanding in power system grids

E.A.R. Theodoro *, R.A.S. Benedito, J.B.A. London Jr., L.F.C. Alberto

Department of Electrical Engineering, School of Engineering of São Carlos, University of São Paulo, Av. Trabalhador São-carlense, nº 400, CEP 13566-590, São Carlos, SP, Brazil

ARTICLE INFO

Article history:
Received 31 March 2011
Received in revised form 29 September 2011
Accepted 10 October 2011
Available online 8 November 2011

Keywords:
Electric power systems
Security analysis
Island formation
Graph theory
Incidence matrices applications

ABSTRACT

In this paper, a new algebraic-graph method for identification of islanding in power system grids is proposed. The proposed method identifies all the possible cases of islanding, due to the loss of a equipment, by means of a factorization of the bus-branch incidence matrix. The main features of this new method include: (i) simple implementation, (ii) high speed, (iii) real-time adaptability, (iv) identification of all islanding cases and (v) identification of the buses that compose each island in case of island formation. The method was successfully tested on large-scale systems such as the reduced south Brazilian system (45 buses/72 branches) and the south-southeast Brazilian system (810 buses/1340 branches).

1. Introduction

The correct identification of physical split in power system grids (islanding) is a challenge task and has become more and more important for security analysis and control in power systems [1,2]. The requirements demanded for these applications are quite different, therefore techniques for identification of island formation in power systems were developed under two well-defined perspectives: security analysis and corrective control (emergency control) [3].

In the context of security analysis, non intentional island formation, due to contingencies, might cause system collapse. Measures to prevent power systems from collapses must be taken as early as possible [4]. As a consequence, the fast identification of possible islanding occurrences is a primary requisite of security assessment tools [5].

In the context of corrective control, intentional islanding is used as a tool for avoiding the total collapse of the system [6,7]. The physical split of a system in small subsystems (islands) is an emergency action that can avoid system collapse in a first moment. The intentional islanding followed by a phase of restoration may be a plausible solution to prevent instabilities and keep the load supply. In this case, the goal of islanding identification is selecting possible corrective actions (intentional islandings), for a given contingency, to avoid a generalized blackout while keeping total or partial load supply [8,9].

A variety of methods were developed for identification of island formation. These methods can be divided in three groups: the first based on chain lists or tables and graph theory (topological methods), the second based on numerical methods, and the third based on compositions of the last two (hybrid methods).

Topological methods usually rely on search algorithms and are usually subjected to either combinatorial explosion or non identification of all possible cases of island formation (most of them work with a limited list of cases). Sasson et al. [10], for example, use chain lists to store the system topology and verify, using sweep algorithms, the connectivity of the grid.

Numerical methods usually explore the structure of the factorized power flow Jacobian matrix. Montagna and Granelli [11], for example, use the factorized load flow Jacobian matrix to detect island formation. One refactorization of the load flow Jacobian matrix is usually required for analyzing each contingency.

Hybrid methods take advantages of both topological and numerical methods. Guler and Gross [12], for example, employ the concept of generalized power transfer distribution factors associated with graph theory to identify, given a set of line outages, a possible island formation. Goderya et al. [13] apply the multiplication of a connectivity matrix to check islanding occurrence given a set of line outages.

It is important to highlight that all the existing methods (topological and numerical), in the current power system literature, check island formation for each individual case of a given list of contingencies. In other words, they do not explore the power grid's

* Corresponding author. Tel.: +55 16 3373 9365; fax: +55 16 3373 9372.
E-mail addresses: edson.theodoro@gmail.com (E.A.R. Theodoro), raphael_usp@hotmail.com (R.A.S. Benedito), jbalj@sc.usp.br (J.B.A. London Jr.), lfcalberto@usp.br (L.F.C. Alberto).

0142-0615/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijepes.2011.10.010

structure to obtain information of a set of contingencies simultaneously.

In this paper, an intrinsic relationship between the power grid's physical topology and its graph representation is explored to develop an algebraic-graph method (hybrid method) for fast identification of islanding. The proposed algebraic-graph method is an achievement towards a faster power system security assessment tool, but the theory developed here is not particular to power systems, and could be applied to different partition-graph problems [14].

The proposed method is based on the triangular factorization of the bus-branch incidence matrix, and it can identify island formation for any given set of branches outages (list of contingencies). Its advantages include easy implementation, high speed and absolute capture of all possible cases of islanding. A special feature of this new method, which makes it very efficient for security analysis applications, is the ability of simultaneously identifying all possible islanding cases, due to the loss of a single equipment (N-1 criterion), with a single factorization of the bus-branch incidence matrix. This feature does not encounter a counterpart in any other of the existing methodologies.

The bus-branch incidence matrix factorization also easily provides the identification of the islands, for any given set of branches outage, via factorization paths. This characteristic makes the proposed method a plausible choice to replace search algorithms such as flood-fill algorithms [15] and integrate topology processors [16], in security assessment applications.

The paper is organized as follows: in Section 2, the proposed method for identification of island formation is developed. In Section 3, the method for identification of islands is presented. Tests and results are shown in Section 4. Conclusions and future perspectives regarding applications of the proposed method are discussed in Section 5.

2. Identification of island formation

In this section, a new method for identification of island formation in power system grids is proposed. Consider a power system grid with $\bar{n}$ buses and $\bar{m}$ branches (transmission lines/transformers).

Definition 1. A power system grid is connected if for every pair of buses there is a path, composed of branches (transmission lines/transformers), which connects them.

A power system grid that is not-connected is composed of isolated connected subsystems. Each isolated connected subsystem is called island.

The elimination of a branch may cause islanding, i.e., the division of a connected system (or subsystem) into two disjoint subsystems (islands). A branch with this property is called critical.

Definition 2. One branch in the power system is critical if and only if its outage causes island formation.

The elimination of a set of branches from the power system may cause islanding too.

Definition 3. A set of branches in the power system is critical if its complete outage causes island formation, while the elimination of any subset of this set of branches does not cause island formation.

The proposed method for islanding identification is based on a triangular factorization of the bus-branch incidence matrix. The bus-branch incidence matrix **H** stores the topology of the system by means of a sparse matrix. Each non-zero element represents the connection of a branch to a bus.

Definition 4. The bus-branch incidence matrix, denoted by **H**, is a $\bar{n} \times \bar{m}$ matrix that stores the topology of the power system grid in its elements according to the following rule:

$$\mathbf{H}_{ij} = \begin{cases} 1 & \text{if the branch } j \text{ is connected to the bus } i; \\ 0 & \text{else.} \end{cases}$$

being $\mathbf{H}_{ij}$ the element in the ith row and jth column of matrix **H**.

Each column of the incidence matrix **H** is associated with a single branch and has exactly two non-zero elements. This pattern will be extensively explored in the proofs of all theorems in this section.

It is remarkable that the bus-branch incidence matrix **H** is also the incidence node matrix of the graph [14], with $\bar{n}$ nodes (buses) and $\bar{m}$ edges (branches), associated with the power system grid. Incidence matrices have been proven to be a powerful tool in several studies of security [12] and control [17] analysis of power systems, once they can deal straight with the system topology.

A redundancy analysis of the branches identifies which of them are critical. Such analysis of redundancy can be performed by a triangular factorization of the incidence matrix **H**. An analysis of the structure of the factorized bus-branch incidence matrix, $\mathbf{H_F}$, indicates all the critical branches, i.e., all the possible causes of islanding for the N-1-criterion, and all the critical set of branches.

Next theorem relates the property of connectivity of a power system grid with the rank of the bus-branch incidence matrix **H**, i.e., the linear dependency among the rows of matrix **H**, in the $\mathbb{Z}_2$ algebra (Appendix A provides an overview of $\mathbb{Z}_2$ algebra). Once each column vector of **H** has exactly two non-zero elements and 1 + 1 = 0 in the $\mathbb{Z}_2$ algebra, then the sum of all $\bar{n}$ row vectors of matrix **H** is equal to zero, i.e., these rows are linearly dependent in the $\mathbb{Z}_2$ algebra. In this manner, $\text{rank}(\mathbf{H}) < \bar{n} \Rightarrow \text{rank}(\mathbf{H}) \leqslant \bar{n} - 1$. However, if the power system grid is connected, then next theorem shows that the rank of **H** is exactly equal to $\bar{n} - 1$.

Theorem 1. *Consider a power system composed of $\bar{n}$ buses and $\bar{m} \geqslant (\bar{n} - 1)$ branches. The following statements are equivalent: (i) the system is connected, (ii) for any $L < \bar{n}$ the sum (in the $\mathbb{Z}_2$ algebra) of any L row vectors of* **H** *is not null and (iii)* $\text{rank}(\mathbf{H}) = \bar{n} - 1$.

Proof of Theorem 1. (i) ⇒ (ii) Suppose for $L < \bar{n}$ the existence of L row vectors in matrix **H** such that the sum (in the $\mathbb{Z}_2$ algebra) of these L row vectors is null. Then each column of $\mathbf{H_{(L)}}$, composed of the selected L row vectors of **H**, contains an even number of non-zero elements.

Then, it is possible to interchange the rows and columns of **H**, such that the first k columns of **H** have two non-zero elements among the L first rows, and so the last $\bar{m} - k$ columns have the L last elements equal to zero. More precisely **H** assumes the form:

$$\mathbf{H} = \left[\begin{array}{c|c} [\mathbf{H}_{(\mathbf{L})}]_{[L\times k]} & [\mathbf{0}]_{[L\times(\bar{m}-k)]} \\ \hline [\mathbf{0}]_{[(\bar{n}-L)\times k]} & [\mathbf{H}_{(\bar{n}-L)}]_{[(\bar{n}-L)\times(\bar{m}-k)]} \end{array} \right]_{[\bar{n}\times\bar{m}]} \tag{1}$$

This results in two partitions of **H**, i.e., the first k branches of the system are connected only to the first L buses, while the last $\bar{m} - k$ branches are connected only to the last $\bar{n} - L$ buses. So the system is a union of two disjointed subsystems, contradicting the initial hypothesis of connectivity (i).

(i) ⇐ (ii) Suppose the system is composed of two disjointed subsystems, one composed of $L < \bar{n}$ buses and k branches and another composed of $\bar{n} - L$ buses and $\bar{m} - k$ branches. So it is possible to show that the incidence matrix H of the entire system assumes the form of (1), with possible changes of order of rows and columns. So the sum (in the $\mathbb{Z}_2$ algebra) of the first L row vectors is null and the result follows.