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# Enhanced cross-entropy method for dynamic economic dispatch with valve-point effects

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ABSTRACT

This paper proposes an enhanced cross-entropy (ECE) method to solve dynamic economic dispatch (DED) problem with valve-point effects. The cross-entropy (CE) method, originated from an adaptive variance minimization algorithm for estimating probabilities of rare events, is a generic approach to combinatorial and multi-extremal optimization. Exploration capability of CE algorithm is enhanced in this paper by using chaotic sequence and the resultant ECE is applied to DED with valve-point effects. The performance of the proposed ECE method is rigorously tested for optimality, convergence, robustness and computational efficiency on a 10-unit test system. Additional test cases with different load patterns and increased number of generators are also solved by ECE. Numerical results show that the proposed ECE approach finds high-quality solutions reliably with faster convergence. It outperforms CE and all the previous approaches.



## 1. Introduction

Power utilities are expected to generate electrical power at minimum cost within the generator and system limits. Economic dispatch (ED) plays a major role in this aspect [1]. Plant operators, to avoid life- shortening of the turbines and boilers, try to keep thermal stress on the equipments within the safe limits. This mechanical constraint is usually transformed into a limit on the rate of change of the electrical output of generators. Such ramp-rate constraints link the generator operation in two consecutive time intervals. ED which includes such inter-temporal dynamic connection is termed as dynamic ED (DED) [2]. The DED has been recognized as not only a more accurate formulation of ED, but also a most challenging optimization problem in power system operation. The DED solution provides the optimal operating trajectories based on the forecasts of system load demand profile. Generating units are then driven along these trajectories by plant controllers to have the lowest operating costs.

Accurate modeling with the inclusion of valve-point loading effects makes the solution space of DED nonconvex with many local minima. Therefore, DED becomes a highly nonlinear and nonconvex optimization problem, which cannot be solved by traditional techniques [3]. Dynamic programming (DP) can solve such type of problems [4], but it suffers from the curse of dimensionality. In recent years, many purebred and hybrid metaheuristic algorithms have been proposed to solve DED with valve-point effects. Mathematical properties such as differentiability, convexity, and linearity are of no concern for these algorithms. Modified differential evolution (MDE) [5] and improved particle swarm optimization (IPSO) [6] are the purebred algorithms that have been applied to DED.

Hybrid algorithms (combination of metaheuristic algorithms and local search procedures i.e. combination of exploration and fine-tuning) have provided significant results for DED with valve-point effects. The constituent algorithms of hybrid methods optimize the problem during different phases of optimization and they are integrated either sequentially or cyclically. Hybrid algorithms like evolutionary programming–sequential quadratic programming (EP–SQP) [7] and improved differential evolution (IDE)-Shor's r-algorithm [8] are examples for sequentially integrated hybrid algorithms, which have been applied to solve DED with valve-point effects. In these algorithms, EP/IDE is used as a base level search; then the fine-tuning is carried out by SQP/Shor's r-algorithm. Even though sequential integration provides better results, it has some drawbacks. First, deciding the point of integration of two algorithms, which has to be specified by the user, is very difficult. At the integration point, there is no guarantee of the favorable state i.e. fine-tuning may be invoked closer to a local-optimum. Secondly, the base algorithm may allow the better regions which are encountered in the earlier iteration stages without fine-tuning.

To ensure fine-tuning at all the stages of optimization, the cyclical hybrid algorithms invoke a deterministic local search procedure whenever the primary heuristic algorithm finds a better solution. Examples of such hybrids are modified hybrid EP–SQP (MHEP–SQP) [9] and deterministically guided PSO (DGPSO-A

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**Nomenclature**

| | |
|---|---|
| $i$ | index for generators; it varies from 1 to $N$ |
| $t$ | index for time intervals; it varies from 1 to $T$ |
| $j$ | index for possible solutions; it varies from 1 to $M$ |
| $k$ | iteration count |
| $k_{\max}$ | maximum iteration count |
| **Sample** | sample matrix |
| $\mathbf{P}^j$ | $j$th sub-matrix of **Sample** ($j$th possible power generation schedule) |
| $P_{i,t}^j$ | $i$, $t$th element (power generation of generator $i$ during interval $t$) of $\mathbf{P}^j$ |
| $M$ | number of individuals (population size or sample size) |
| $\mathbf{v}_{i,t}$ | parameter vector for generator $i$ during interval $t$ |
| $\mu_{i,t}/\sigma_{i,t}$ | mean/std of all the $M$ power generations of generator $i$ during interval $t$ |
| $PF_T(\mathbf{P}^j)$ | penalized objective function value for $\mathbf{P}^j$ |
| $F_T^{\text{best}}(k)$ | best objective function (generation cost) value at iteration $k$ |
| $\mathbf{P}^{\text{best}}(k)$ | best power generation schedule at iteration $k$ |
| $F_T^{\text{opt}}$ | optimum objective function value |
| $\mathbf{P}^{\text{opt}}$ | optimum power generation schedule |
| $\rho$ | rarity parameter |
| $\alpha/\beta$ | smoothing parameter for mean/std |
| $\beta(k)$ | value of $\beta$ at iteration $k$ |
| $z(k)$ | value of the chaotic sequence at iteration $k$ |
| $\beta_z(k)$ | value of the chaotic sequence modulated $\beta$ at iteration $k$ |
| $\mathbf{P}^{\text{elites}}$ | elite set of power generation schedule |
| $\tilde{\mathbf{v}}_{i,t}$ | estimated $\mathbf{v}_{i,t}$ |
| $\tilde{\mu}_{i,t}/\tilde{\sigma}_{i,t}$ | estimated $\mu_{i,t}/\sigma_{i,t}$ |
| $P_{i,t}^{\text{elites}}$ | $i$, $t$th element of the all the elites |
| $\mathbf{v}$ | parameter vector consists of mean ($\mu$) and std ($\sigma$) |
| $\tilde{\mathbf{v}}$ | estimated $\mathbf{v}$ consists of estimated mean ($\tilde{\mu}$) and estimated std ($\tilde{\sigma}$) |
| $f(.;\mathbf{v})$ | pdf with parameter vector $\mathbf{v}$ |

hybrid of PSO and SQP) [10]. The SQP fine-tunes the solution obtained by EP/PSO algorithm. Then the primary algorithm i.e. EP/PSO takes the solution of SQP as a guide and optimizes the problem further. Since SQP is invoked during the favorable state there are more possibilities to get better solution. Therefore, the cyclic integration used in MHEP–SQP and DGPSO is better than sequential integration. However, the cyclical hybrid algorithms consume large CPU time due to the often invoking of SQP. Another weakness is that SQP may wrongly guide the heuristic algorithm towards a local-optimum. Once the SQP finds a better solution (may be a local-optimum), the primary algorithm is attracted towards it.

The preceding discussion reveals that there is a need of simple yet powerful algorithms to solve DED with valve-point effects. In this context, this paper proposes an enhanced cross-entropy (ECE) method to solve the DED problem with valve-point effects. Cross-entropy (CE), proposed by Rubinstein [11,12], is an innovative metaheuristic approach. CE was originally proposed as an adaptive variance minimization algorithm for estimating rare event probabilities. Later, it has been applied to complex combinatorial optimization problems [13–19]. In this paper, chaotic sequence is used to enhance the performance of CE and the resultant ECE algorithm is applied to solve DED with valve-point effects. Recent research trend is adopting chaotic sequences instead of random ones and very promising results have been obtained in many engineering applications [20]. The application of chaotic sequences in CE results in a powerful strategy, which has improved global searching ability and capacity to escape from local minima.

To validate the proposed ECE method in solving DED with valve-point effects, it is tested on a system of 10 units with 24 h scheduling span. Also, ECE has been tested with different load patterns and increased number of generators. Performance comparisons with existing methods on solution optimality, consistency and execution time are presented. Test results confirm that the proposed ECE is more consistent in giving lower generation cost with shorter execution time than previous approaches. ECE, also, outperforms CE in solving DED.

## 2. DED problem formulation

DED aims in economically scheduling the online generators over the dispatch horizon such that the forecasted load profile is met. While doing so, system and generator constraints must be satisfied. Accordingly, DED can be formulated as an optimization problem.

### 2.1. Objective function

The objective of DED is to minimize the fuel cost of all the online generators over the given dispatch horizon. Mathematically, it can be stated as:

$$\text{minimize } F_T = \sum_{t=1}^{T} \sum_{i=1}^{N} F_i(P_{i,t}) \tag{1}$$

where $F_T$ is total fuel cost or generation cost ($); $N$ is the number of generating units; $T$ is the number of intervals in the scheduling horizon; $P_{i,t}$ is the real power generation of generating unit $i$ during subinterval $t$ (MW) and $F_i()$ is the fuel-cost function of generating unit $i$. To improve the accuracy of DED formulation, the valve-point effect (ripple like effect in the heat rate curve due to sequential valve opening process) is included by superimposing the basic quadratic fuel-cost characteristics with the rectified sinusoidal component as follows:

$$F_i(P_{i,t}) = a_i P_{i,t}^2 + b_i P_{i,t} + c_i + |e_i \sin(f_i(P_i^{\min} - P_{i,t}))| (\$/h) \tag{2}$$

where $a_i$, $b_i$, $c_i$ are the cost coefficients and $e_i, f_i$ are the valve-point effect coefficients of generator $i$; $P_i^{\min}$ and $P_i^{\max}$ are the minimum and maximum power generation limits of generator $i$, respectively.

### 2.2. Constraints

(a) Demand-supply balance

$$\sum_{i=1}^{N} P_{i,t} = P_{Dt} + P_{Lt} \tag{3}$$

where $P_{Dt}$ is the total load in the system during subinterval $t$ (MW) and $P_{Lt}$ is the network loss during subinterval $t$ (MW), which is determined using **B**-matrix loss formula [1].

$$P_{Lt} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_{i,t} B_{ij} P_{j,t} + \sum_{i=1}^{N} B_{0i} P_{i,t} + B_{00} \tag{4}$$

where $B_{ij}$ is the element of loss coefficient square matrix of size $N$; $B_{0i}$ is the element of loss coefficient vector of length $N$ and $B_{00}$ is the loss coefficient constant.

(b) Real power generation limits

$$P_i^{\min} \le P_{i,t} \le P_i^{\max} \tag{5}$$