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Short Communication

Linearized voltage stability index for wide-area voltage monitoring and control

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ABSTRACT

This letter addresses a linearized local voltage stability index based on wide-area measurement systems. The measurements allow reducing the problem of assessing the stability of a large system with *n* buses to the study of the local stability of *n* load buses. The proposed index includes the dynamic model of the induction motor and is based on the determinant of the equivalent load state matrix. Simulation results show that the proposed index is computationally light, fairly linear and highly accurate.

1. Introduction

On-line voltage stability preventive schemes based on wide-area measurement systems (WAMS) are currently under study for improving system reliability and robustness [1–4]. This letter addresses the prediction of voltage instability and collapse of real size systems based on a local voltage stability index. The index is computed based on available PMU measurements and on load parameters.

About half of the total electric power used in the industrial processes is consumed by induction motors. Nevertheless, the literature is scarce on the proposal of voltage stability indices that include the dynamic load models [5,6]. In this letter, we propose a linearized local voltage stability index considering dynamic load characteristics. The index is based on the state matrix determinant of a measurement-based load composed of an induction motor in parallel with a constant impedance. The proposed index shows better performances and features than a variety of local voltage stability indices proposed in the literature [1–3,5,7].

2. Induction motor dynamic model

The dynamic model of the induction motor is as follows:

$$\frac{ds}{dt} = \frac{1}{T_j}(T_M - T_E) \tag{1}$$

$$\frac{de'_x}{dt} = \frac{1}{T'_{d0}}(-e'_x + (X - X')i_y + T'_{d0}e'_y s \cdot 2\pi f_0) \tag{2}$$

$$\frac{de'_y}{dt} = \frac{1}{T'_{d0}}(-e'_y + (X - X')i_x - T'_{d0}e'_x s \cdot 2\pi f_0) \tag{3}$$

where the definitions of most parameters can be found in Ref. [5]. Furthermore, $T_M = K_L[\alpha + (1-\alpha)(1-s)^P]$ is the mechanical torque; s is the rotor slip; T_j and T'_{d0} are the rotor time constant and transient open-circuit time constant, respectively; e'_x and e'_y are the real and reactive part of transient electromotive force $e\prime$, respectively; and T_M and T_E are the load torque and the electromagnetic torque of induction motor, respectively.

The linearization of (1)–(3) at an equilibrium point leads to:

$$\begin{bmatrix} \Delta\dot{s} \\ \Delta\dot{e}'_x \\ \Delta\dot{e}'_y \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} \Delta s \\ \Delta e'_x \\ \Delta e'_y \end{bmatrix} = A \begin{bmatrix} \Delta s \\ \Delta e'_x \\ \Delta e'_y \end{bmatrix} \tag{4}$$

where:

$$A_{11} = \frac{-PK_L(1-\alpha)(1-s)^{P-1}}{T_j}, \quad A_{12} = \frac{R_1(2e'_x - V_x) - X'V_y}{T_j(R_1^2 + X'^2)}, \quad A_{13} = \frac{R_1(2e'_y - V_y) + X'V_x}{T_j(R_1^2 + X'^2)},$$
$$A_{21} = 2\pi f_0 e'_y,$$
$$A_{22} = -\frac{1}{T'_{d0}}\left(1 + \frac{X'(X - X')}{R_1^2 + X'^2}\right), \quad A_{23} = \frac{R_1(X - X')}{T'_{d0}(R_1^2 + X'^2)} + 2\pi f_0 s, \; A_{31} = -2\pi f_0 e'_x,$$
$$A_{32} = -\left(\frac{R_1(X - X')}{T'_{d0}(R_1^2 + X'^2)} + 2\pi f_0 s\right),$$
$$A_{33} = -\frac{1}{T'_{d0}}\left(1 + \frac{X'(X - X')}{R_1^2 + X'^2}\right).$$

where R_1 and X_1 are the stator resistance and reactance, respectively; R_2 and X_2 are the rotor resistance and reactance, respectively; $X_\mu = T'_{d0}R_2 \cdot 2\pi f_0$ is the magnetizing reactance of induction motor. Then, the transient reactance X' and reactance X are defined: $X' = X_1 + X_2//X_\mu$, $X = X_1 + X_\mu$.

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Load voltage $\dot{V}$, transient electromotive force $\dot{e}'$ and current $\dot{i}$ satisfy the following equation:

$$\dot{V} = \dot{e}' - (R_1 + jX')\dot{i} \tag{5}$$

In general, changes of stator resistance R_1 have little impacts on the small-signal stability results. Ref. [8] shows that eigenvalues of induction motor Jacobian matrix change very little as R_1 changes from 0.1 to 0. Then assuming $R_1 = 0$, we have $P_I = T_M = T_E$ (where, P_I is the active power of induction motor). Integrating Eqs. (2), (3), and (5) and the power equation of induction motor, we have:

$$e'_{x0} - (X - X')i_{y0} - T'_{d0}e'_{y0}s_0 \cdot 2\pi f_0 = 0 \tag{6}$$
$$e'_{y0} + (X - X')i_{x0} + T'_{d0}e'_{x0}s_0 \cdot 2\pi f_0 = 0 \tag{7}$$
$$e'_{x0} - R_1 i_{x0} + X' i_{y0} - V_{x0} = 0 \tag{8}$$
$$e'_{y0} - R_1 i_{y0} - X' i_{x0} - V_{y0} = 0 \tag{9}$$
$$P_I + (V_{x0}i_{x0} + V_{y0}i_{y0}) = 0 \tag{10}$$

Once V_{x0}, V_{y0}, P_I are obtained with PMU measurements, $(s_0, e'_{x0}, e'_{y0}, i_{x0}, i_{y0},)$ can be calculated through solving Eqs. (6)–(10).

The accuracy of the load model has great effects on voltage stability. Generally speaking, a load model composed of an induction motor in parallel with a constant impedance is sufficient for stability analysis. In this letter, we assume a fixed proportion between the induction motor load and the constant impedance load. When used in a power system, composite load modeling based on measurements can be used instead [9]. Typical induction motor parameters used in this letter are shown in Table 1.

3. Linearized motor voltage stability index

The properties of the state matrix A are used to assess the stability of each bus. As it will be explained below, generator models, reactive power limits, transmission line saturation, etc. are implicitly taken into account by the measurements obtained from the WAMS, and thus are not directly modelled.

At a generic time t_i, the proposed motor voltage stability index (MVSI) is defined as follows:

$$\mathrm{MVSI}_i = |\det(A_i)| \tag{11}$$

where, A_i is the Jacobian matrix at a given time t_i and $\det(\cdot)$ denotes the determinant of A_i. Observe that A_i depends on the bus voltage and the load power, which are obtained by PMU measurements.

The MVSI can be easily linearized, as follows [7]:

$$\mathrm{LMVSI}_i = \frac{\mathrm{MVSI}_i}{|d(\mathrm{MVSI}_i)/d\lambda_i|} \tag{12}$$

Here, λ_i is the loading factor at a given time t_i:

$$\lambda_i = \frac{P_L^{t_i} - P_L^{t_0}}{P_L^{t_0}} \tag{13}$$

Where $P_L^{t_i}$ and $P_L^{t_0}$ are the active power at the time t_i and t_0, respectively.

To compute the critical loading margin of each bus (i.e. the value of λ for which LMVSI = 0), one needs to know how the bus voltage varies as a function of the load powers. In the literature, this information is obtained by determining the Thevenin equivalent of the network at the load bus [1–3,5]. However, this step can be time consuming, especially for on-line applications.

For this reason, in this letter, system equivalents are not computed. Instead of computing the equivalents, we use measured voltages and powers at different times, as follows. Let us assume that the load at bus k is varied in the time interval between t_{i-1} and t_i. Assuming that the index is fully linear, the predicted critical loading margin $\lambda_k^{\mathrm{crit}}$ is:

$$\lambda_k^{\mathrm{crit}} = \lambda_{i-1,k} - \frac{\mathrm{LMVSI}_{i-1,k}(\lambda_{i,k} - \lambda_{i-1,k})}{\mathrm{LMVSI}_{i,k} - \mathrm{LMVSI}_{i-1,k}} \tag{14}$$

Since there is one critical λ per load bus, the resulting system-wide loading margin is defined by the minimum critical loading margin predicted for all buses, as follows:

$$\lambda^{\mathrm{crit}} = \min_k \left\{\lambda_k^{\mathrm{crit}}\right\} \tag{15}$$

The proposed LMVSI has the following relevant features:

3.1. No need of computing network equivalents

The network is implicitly taken into account by WAMS measurements at different time steps.

3.2. Full linearity

As shown in the case study section, the proposed LMVSI is fully linear for all values of the loading margin λ.

3.3. Low computational burden

The determinant of a 3×3 matrix is computationally more efficient than the linearized minimum eigenvalue index (LMEI) and the linearized minimum singular value index (LMSVI) that are described in Ref. [7]. Furthermore, $\lambda_k^{\mathrm{crit}}$ can be computed at each time t_i with a minimum computational effort.

4. Case studies

Fig. 1 shows the behavior of the LMVSI for the New England 39-bus system. The index is computed off-line for several values of λ. For the sake of comparison, the LMEI and the LMSVI are also depicted. Observe that the LMEI does not show a fully linear behavior.

Fig. 2 shows the behavior of the LMVSI in different scenarios, including full system, $N-1$ contingency and $N-2$ contingency.

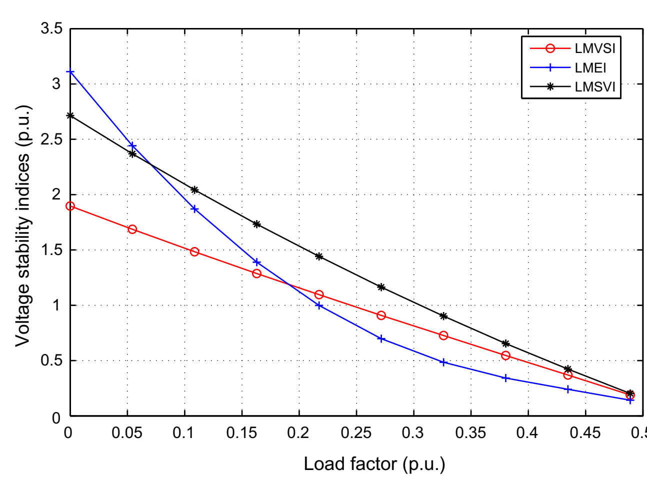

Fig. 1. Comparison of LMSVI and LMEI and the proposed LMVSI for the 39-bus system.

Table 1
Typical induction motor parameters.

$T'_{d0} = 0.576$	$R_1 = 0$
$T_j = 2.0$	$R_2 = 0.02$
$P = 2$	$X_1 = 0.18$
$\alpha = 0.15$	$X_2 = 0.12$