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# Interval analysis applied to the maximum loading point of electric power systems considering load data uncertainties

L.E.S. Pereira [a,1], V.M. da Costa [b,*]

[a] Department of Electrical Engineering, Federal Institute of Minas Gerais, Rua Padre Alberico, 440, São Luiz, 35570-000 Formiga – MG, Brazil
[b] Department of Electrical Engineering, Federal University of Juiz de Fora, Campus Universitário, Bairro Martelos, 36036-330 Juiz de Fora – MG, Brazil

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ABSTRACT

This paper proposes a simple and efficient power flow method to calculate, in an interval manner, the main variables corresponding to the maximum loading point, under load data uncertainties. The resulting interval nonlinear system of equations is solved using Krawczyk method. The proposed methodology is implemented in the Matlab environment using the Intlab toolbox. Results are compared with those obtainable by Monte Carlo simulations. IEEE 30 bus system and a South-southeastern Brazilian network are used to validate the proposed methodology.



## 1. Introduction

Power flow [1,2] is the most frequently performed study in electric power systems, and deals with the calculation of voltages and line flows, in a large sparse electrical network, for a given load and generation schedule. The conventional power flow solution comprises power equations expressed in terms of polar or rectangular voltage coordinates. Over the last years, the current injection power flow has been applied to different electric power system areas and remarkable results have been published in literature [3–6].

Voltage stability has been one of the major concerns of power system operators and planners for the last years. The continuous load increase, allied to the lack of investments in transmission and generation, has led systems to operate very close to their limits. Voltage stability has been a subject widely investigated [7]. Its static analysis can be assessed through continuation power flow and point of collapse methods.

The voltage profile is obtained through successive power flow solutions by simulating load changes. However, the voltage profile cannot be traced completely, by using only the conventional power flow, because the Jacobian matrix becomes singular at the maximum loading point (MLP). The continuation method is applied to power flow equations to overcome this drawback. A brief history is presented next. In [8], a mathematical model for the continuation power flow using either additional load change, or voltage magnitude, as continuation parameters is presented. A tool for evaluating nonlinear effects on power system states due to branch admittance/impedance variations is presented in [9]. A continuation three-phase power flow is proposed in [10]. In [11], a fuzzy continuation power flow is developed with the objective of simultaneously handling uncertainties in load parameters and bus injections parameters.

The objective of point of collapse method is to iteratively calculate the maximum loading point of electric power systems without tracing the continuation curves. A brief history is presented next. Ref. [12] describes an extension of the point of collapse developed for ac systems studies to the determination of saddle-node bifurcations in power systems, including high voltage direct current transmission. Ref. [13] describes the implementation of both point of collapse and continuation methods for computation of voltage collapse in large ac/dc systems. Ref. [14] calculates the maximum loadability using interior point nonlinear optimization method.

On the other hand, input data are subject to uncertainty. For instance, all loads are provided by measurement devices which are frequently inaccurate. One of the approaches used for taking into account the effects of errors on numerical computation is the interval analysis. It is based on interval operations including interval arithmetic. The technique calculates the interval between the upper and lower bounds regarding variables under uncertainty. To yield solutions in a mathematical sense and to express the variable under uncertainty as an interval variable are the main

* Corresponding author. Tel.: +55 32 21023461; fax: +55 32 21023442.
*E-mail addresses:* eduardo.pereira@ifmg.edu.br (L.E.S. Pereira), vander.costa@ufjf.edu.br (V.M. da Costa).
[1] Tel.: +55 37 91048816.

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advantages. The main limitation is sometimes to overestimate the interval between the upper and lower bounds. Therefore, loads and other parameters can be characterized not by a single number, but rather by a range of real values or a real interval [15–17]. Other approaches for dealing with uncertainties are the probabilistic power flow [18–21] and the fuzzy power flow [22–25].

Ref. [26] presents a model of power flow under uncertainty by incorporating interval arithmetic into the current injection formulation. No control devices are considered in the power flow problem and the interval solutions yielded by methodology refer to the nominal operating point. The main objective of this paper is to extend the methodology developed in [26] in order to calculate in an interval form, under load data uncertainty, not only the maximum loading point, but also the main variables corresponding to this point, such as voltage magnitudes, phase angles, active and reactive power generations, active and reactive line flows and losses. Reactive power generation limits at PV buses and voltage magnitudes limits at PQ buses are considered.

The notations adopted in the paper are the conventional ones whenever possible. Matrices are shown in bold. The over scripts $d$ and $i$ refer to deterministic and interval quantities, respectively.

## 2. Brief review on interval technique [26]

An interval number $[x_1, x_2]$ is the set of real numbers x such that $x_1 \leqslant x \leqslant x_2$. $x_1$ is the infimum and $x_2$ is the supremum. The interval $X$ is defined by $X = [x_1, x_2] = \{\tilde{x} \in \mathbb{R} | \mathbb{x}_1 \leqslant \tilde{\mathbb{x}} \leqslant \mathbb{x}_2\}$.

One of the most used approaches for solving a set of nonlinear equations is the Krawczyk method. Let $f$ be a nonlinear function such that $f(x) = 0$. The Krawczyk operator is given by

$$K(x^{(h)}, X^{(h)}) = x^{(h)} - \mathbf{C}f(x^{(h)}) + [\mathbf{I} - \mathbf{CJ}(X^{(h)})](X^{(h)} - x^{(h)}) \tag{1}$$

where $x$ is the midpoint of interval $X$, $\mathbf{J}$ the Jacobian matrix, $\mathbf{I}$ the identity matrix, $\mathbf{C}$ the preconditioning matrix given by the midpoint inverse of $\mathbf{J}(X)$ and h is the iteration number.

The interval solution is given by.

$$X^{(h+1)} = X^{(h)} \cap K(x^{(h)}, X^{(h)}) \tag{2}$$

Eq. (2) means that the interval Krawczyk method provides the solution through the intersection of two interval sets. The iterative process converges when $|X^{(h+1)} - X^{(h)}| \leqslant \Im$.

Let $X = [x_1, x_2]$ and $Y = [y_1, y_2]$ be two intervals. The interval operations in Eqs. (1) and (2) are given by

$$X + Y = [x_1 + y_1, x_2 + y_2] \tag{3}$$

$$X - Y = [x_1 - y_2, x_2 - y_1] \tag{4}$$

$$X \cdot Y = [\min(x_1y_1, x_1y_2, x_2y_1, x_2y_2), \max(x_1y_1, x_1y_2, x_2y_1, x_2y_2)] \tag{5}$$

$$\frac{X}{Y} = \left[\min\left(\frac{x_1}{y_1}, \frac{x_1}{y_2}, \frac{x_2}{y_1}, \frac{x_2}{y_2}\right), \max\left(\frac{x_1}{y_1}, \frac{x_1}{y_2}, \frac{x_2}{y_1}, \frac{x_2}{y_2}\right)\right] \tag{6}$$

$$\begin{aligned} & X \cap Y = [\max\{x_1, y_1\}; \min\{x_2, y_2\}]; \\ & \text{if } \max\{x_1, y_1\} < \min\{x_2, y_2\} \\ & \text{if } \min\{x_2, y_2\} > \max\{x_1, y_1\} \\ & \text{then } X \cap Y = 0 \end{aligned} \tag{7}$$

## 3. Interval power flow solution at maximum loading point – proposed method

### 3.1. Initial Considerations

This paper proposes a new methodology in order to calculate in an interval form, under load data uncertainty, the maximum loading point and the main variables corresponding to this point, such as voltage magnitudes, phase angles, active and reactive power generations, active and reactive line flows and losses. The power flow problem is modeled through current injection equations written in rectangular voltage coordinates.

### 3.2. Solution methodology

The interval power flow solution at MLP, denoted by IPFS-MLP, can be summarized in the following steps:

Step 1: Run the deterministic PSAT (Power System Analysis Toolbox) program [27] to calculate the MLP and all deterministic variables associated with this point. The configurations adopted to run the continuation power flow in PSAT program are: corrector step tolerance = $10^{-5}$, flow tolerance = 0.01; step size control = 0.005 and maximum number of points = 5000. Besides, representation of control devices is also activated.

Step 2: Active and reactive load variations, in the base case, at a generic bus $k$ are given by

$$P^i_{d_k} = [P^d_{d_k}(1 - \alpha_{P_k}), P^d_{d_k}(1 + \alpha_{P_k})] \tag{8}$$

$$Q^i_{d_k} = [Q^d_{d_k}(1 - \alpha_{Q_k}), Q^d_{d_k}(1 + \alpha_{Q_k})] \tag{9}$$

where $\alpha_{P_k}$ and $\alpha_{Q_k}$ are factors which denote active and reactive load variations.

Step 3: A new variable $\gamma$ is employed to simulate load and generation changes. Therefore, the real and imaginary components of interval current mismatches [26] must be calculated by considering this extra variable. Thus

$$\Delta I^i_{r_k} = I^d_{r_k} - \frac{(P^i_{g_k} - P^i_{d_k})(1 + \gamma^d)V^d_{r_k} + (Q^i_{g_k} - Q^i_{d_k})(1 + \gamma^d)V^d_{m_k}}{(V^d_k)^2} \tag{10}$$

$$\Delta I^i_{m_k} = I^d_{m_k} - \frac{(P^i_{g_k} - P^i_{d_k})(1 + \gamma^d)V^d_{m_k} - (Q^i_{g_k} - Q^i_{d_k})(1 + \gamma^d)V^d_{r_k}}{(V^d_k)^2} \tag{11}$$

where $V_k$ is the voltage magnitude at MLP; $\Delta I_{r_k} + j\Delta I_{m_k}$ is the complex current mismatch at MLP; $P_{g_k} + jQ_{g_k}$ is the generated complex power in the base case; the under script $k$ denotes the bus. Current mismatches are calculated only once.

Step 4: Interval voltages are initialized by using the deterministic voltage profile at MLP as midpoint. In order to improve the interval initial conditions, the radius is given by

$$\begin{bmatrix} \Delta V^i_r \\ \Delta V^i_m \end{bmatrix} = (J^d)^{-1} \begin{bmatrix} \Delta I^i_m \\ \Delta I^i_r \end{bmatrix} \tag{12}$$

Therefore

$$V^i_{r_k} = V^d_{r_k} + \Delta V^i_{r_k} \tag{13}$$

$$V^i_{m_k} = V^d_{m_k} + \Delta V^i_{m_k} \tag{14}$$

Since the deterministic current injection Jacobian matrix in Eq. (12), denoted by $J^d$, is singular at MLP, the strategy adopted in this paper is to calculate this matrix at a point close to MLP. For example, if the PSAT program requires $n$ points to calculate the MLP, then it is evaluated by using all power flow variables corresponding to the $(n - 2)$ point.