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A widely linear least mean phase algorithm for adaptive frequency estimation of unbalanced power systems

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ABSTRACT

A robust technique for online estimation of the fundamental frequency of both balanced and unbalanced three-phase power systems is proposed. This is achieved by introducing a widely linear least mean phase (WL-LMP) frequency estimator, based on Clarke's transformation and widely linear complex domain modelling. The proposed method makes use of the full second-order information within the complex-valued system voltage, making it possible to eliminate otherwise unavoidable oscillations in frequency estimation. In this way, the WL-LMP inherits the advantages of the phase-only approach, such as its high angle estimation accuracy and immunity to voltage and harmonics variations, while accounting for the noncircularity of Clarke's voltage in unbalanced conditions. Simulations over a range of unbalanced conditions, including those caused by voltage sags and higher order harmonics, and case studies for real-world unbalanced power systems support the analysis.

1. Introduction

Frequency is a key variable in power quality control, as its fluctuations reflect the dynamic balance between power generation and load consumption [1]. The need for its accurate estimation is even more highlighted through current trends for distributed generation, which require perfect system synchrony is needed to connect microgrids and regulate islanding. In those scenarios, some fluctuating loads, such as electric arc furnaces, adjustable speed drives (ASDs), and nonlinear electric devices, are sources of harmful voltage fluctuations, higher order harmonics, amplitude and phase noise, and system frequency deviation [2].

To deal with these issues in a timely and efficient way, fast and accurate frequency estimation has attracted much research effort. A variety of linear and nonlinear architectures and the associated signal processing algorithms have been developed for this purpose, including zero crossing techniques [3,4], discrete Fourier transform (DFT) based algorithms [5,6], phase-locked loops (PLL) [7,8], complex least mean square (CLMS) adaptive filters [9,10], recursive Newton-type algorithms [11], and Kalman filters [12,13]. Among these, adaptive approaches based on the minimisation of mean square error have proved very useful, owing to their simple structure, computational efficiency and stability, and robustness in the presence of noise and harmonic distortions.

There are a number of applications where the mean square error (MSE) criterion is not the most intuitive solution, particularly when the information of interest is contained predominantly in either the amplitude or phase of a complex signal. Such is the case with frequency estimation in power systems, where the desired information is primarily in the complex phasor, therefore phase error in the estimation is more critical than the amplitude error, and hence the standard MSE based CLMS is not best equipped to deal with predominantly phase error. To that cause, the recent least mean phase (LMP) algorithm employs an optimisation criterion based on the phase error [14], and has proven beneficial in communications applications (DS-CDMA receivers), where the relevant information is in the phase of the transmitted signals rather than in the magnitude. A continuous-time version of this algorithm has recently been applied to estimate the power system frequency [15], and its superiority over the standard CLMS was justified by the fact that the instantaneous frequency estimation is derived from the well-established phase angle evolution.

Although current adaptive filtering based frequency estimation algorithms are second order optimal under normal 'balanced' power system conditions, and also in noisy environments and in the presence of high order harmonics and frequency deviations, they suffer from performance degradation under unbalanced

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voltage conditions. These occur in the case of different amplitudes of the three phase voltages, or under a voltage sag in one or two phases. Their inadequacy can be explained by the fact that current adaptive filters employ the standard linear model to carry out the phase angle estimation, thus accounting only for the 'positive sequence' (with positive frequency) in the $\alpha\beta$ transform, however, the 'negative sequence' introduced by the system imbalance results in an inevitable estimation error oscillating at twice the system frequency [16].

To eliminate the bias and steady state oscillations encountered by the LMP algorithm under unbalanced system conditions, in this work, we embark upon the analysis in [17–19] and introduce the widely linear LMP (WL-LMP) algorithm, which caters for the noncircular nature of the complex-valued system voltage under the unbalanced system conditions. This allows us to rectify the issue of phase angle bias exhibited by the strictly linear LMP algorithm in unbalanced system conditions, while for balanced systems we show that WL-LMP degenerates into the strictly linear LMP.

This paper is organised as follows. In Section 2, an overview of widely linear estimation is provided. The modelling of unbalanced three-phase power system is addressed in Section 3. In Section 4, an overview of the standard LMP frequency estimator and its suboptimality under unbalanced power systems are discussed. The widely linear model, which is second order optimal for the generality of complex-valued signals, and the proposed unbiased widely linear LMP (WL-LMP) frequency estimator are introduced in Section 5. Section 6 presents the stability analysis of WL-LMP. In Section 7, simulations over a range of unbalanced and distorted conditions, including voltage sags, higher order harmonics, and real-world unbalanced power systems are provided to illustrate the unbiasedness of the proposed WL-LMP frequency estimator and its advantages over the mean squared error based widely linear complex least mean square (WL-CLMS) frequency estimator [17]. Finally, Section 8 concludes the paper.

2. Widely linear modelling

Consider a real-valued conditional mean square error (MSE) estimator $\hat{y} = E[y|\mathbf{x}]$, which estimates the signal y in terms of another observation $\mathbf{x}$. For zero mean, jointly normal y and $\mathbf{x}$, the optimal solution is given by the linear model

$$\hat{y} = \mathbf{x}^T\mathbf{w} \tag{1}$$

where $\mathbf{w} = [w_1, \ldots, w_L]^T$ is the vector of fixed filter coefficients, $\mathbf{x} = [x_1, \ldots, x_L]^T$ the regressor vector, and $(\cdot)^T$ the vector transpose operator.

In the complex domain, since both the real and imaginary parts of complex variables are real, we have

$$\begin{aligned}\Re(\hat{y}) &= E[\Re(y)|\Re(\mathbf{x}),\Im(\mathbf{x})]\\ \Im(\hat{y}) &= E[\Im(y)|\Re(\mathbf{x}),\Im(\mathbf{x})]\end{aligned} \tag{2}$$

and $\hat{y} = E[\Re(\hat{y})|\Re(\mathbf{x}),\Im(\mathbf{x})] + \jmath E[\Im(y)|\Re(\mathbf{x}),\Im(\mathbf{x})]$, where the operators $\Re(\cdot)$ and $\Im(\cdot)$ extract respectively the real and imaginary parts of a complex variable. Upon substituting $\Re(\mathbf{x}) = (\mathbf{x}+\mathbf{x}^*)/2$ and $\Im(\mathbf{x}) = (\mathbf{x}-\mathbf{x}^*)/2\jmath$, we arrive at

$$\hat{y} = E[\Re(y)|\mathbf{x},\mathbf{x}^*] + \jmath E[\Im(y)|\mathbf{x},\mathbf{x}^*] = E[y|\mathbf{x},\mathbf{x}^*] \tag{3}$$

leading to the widely linear estimator

$$\hat{y} = \mathbf{h}^T\mathbf{x} + \mathbf{g}^T\mathbf{x}^* = \mathbf{x}^T\mathbf{h} + \mathbf{x}^H\mathbf{g} \tag{4}$$

where $\mathbf{h}$ and $\mathbf{g}$ are complex-valued coefficient vectors. Such a widely linear estimator is optimal for the generality of complex signals. From (4), it is apparent that the covariance matrix $\mathbf{C_{xx}} = E[\mathbf{xx}^H]$ alone does not have sufficient degrees of freedom to describe full second-order statistics [20], and in order to make use of all the available statistical information, we also need to consider the pseudo-covariance matrix $\mathbf{P_{xx}} = E[\mathbf{xx}^T]$. Processes whose second-order statistics can be accurately described by only the covariance matrix, that is with $\mathbf{P_{xx}} = \mathbf{0}$, are termed second-order circular (or proper), such signals have rotation-invariant distributions $\mathcal{P}[\cdot]$ for which $\mathcal{P}[\mathbf{z}] = \mathcal{P}[\mathbf{z}e^{j\theta}]$ for $\theta \in [0, 2\pi)$. However, in order to cater for second order noncircular (or improper) signals (with rotation dependent distributions), the widely linear model in (4) should be employed, whereby the regressor vector is produced by concatenating the input vector $\mathbf{x}$ with its conjugate $\mathbf{x}^*$, to give an augmented $(2L \times 1)$-dimensional input vector $\mathbf{x}^a = [\mathbf{x}^T, \mathbf{x}^H]^T$, together with the augmented coefficient vector $\mathbf{w}^a = [\mathbf{h}^T, \mathbf{g}^T]^T$. The corresponding $(2L \times 2L)$-dimensional augmented covariance matrix then becomes

$$\mathbf{C}^a_{\mathbf{xx}} = E[\mathbf{x}^a\mathbf{x}^{aH}] = E\begin{bmatrix}\mathbf{x}\\ \mathbf{x}^*\end{bmatrix}[\mathbf{x}^H\mathbf{x}^T] = \begin{bmatrix}\mathbf{C_{xx}} & \mathbf{P_{xx}}\\ \mathbf{P}^*_{\mathbf{xx}} & \mathbf{C}^*_{\mathbf{xx}}\end{bmatrix} \tag{5}$$

and contains the full second order statistical information [21–23].

3. Unbalanced three-phase power systems

The three-phase voltages of a power system in a noise-free environment can be represented in a discrete time form as

$$\begin{aligned}v_a(k) &= V_a\cos(\omega k\Delta\mathrm{T} + \phi)\\ v_b(k) &= V_b\cos\left(\omega k\Delta\mathrm{T} + \phi - \frac{2\pi}{3}\right)\\ v_c(k) &= V_c\cos\left(\omega k\Delta\mathrm{T} + \phi + \frac{2\pi}{3}\right)\end{aligned} \tag{6}$$

where V_a, V_b, V_c are the peak values of each fundamental voltage component at time instant k, $\Delta\mathrm{T} = \frac{1}{f_s}$ is the sampling interval where f_s is the sampling frequency, ϕ is the initial phase, and $\omega = 2\pi f$ is angular frequency of the voltage signal, with f being the system frequency. For analysis purpose, the time-dependent three-phase voltage in (6) is routinely transformed by the orthogonal $\alpha\beta$ transformation matrix [24] into a zero-sequence v_0 and the direct and quadrature-axis components, v_α and v_β, as

$$\begin{bmatrix}v_0(k)\\ v_\alpha(k)\\ v_\beta(k)\end{bmatrix} = \sqrt{\frac{2}{3}}\begin{bmatrix}\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\\ 1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}v_a(k)\\ v_b(k)\\ v_c(k)\end{bmatrix} \tag{7}$$

where the factor $\sqrt{2/3}$ ensures that the system power is invariant under this transformation. In balanced system conditions, $V_a(k)$, $V_b(k)$, $V_c(k)$ are identical, giving $v_0(k) = 0$, $v_\alpha(k) = A\cos(\omega k\Delta\mathrm{T} + \phi)$ and $v_\beta(k) = A\cos\left(\omega k\Delta\mathrm{T} + \phi + \frac{\pi}{2}\right)$. The amplitude, $A = \frac{\sqrt{6}(V_a+V_b+V_c)}{6}$, is constant while $v_\alpha(k)$ and $v_\beta(k)$ represent the orthogonal coordinates of a point whose position is time variant at a rate proportional to the system frequency. In practise, normally only the v_α and v_β are used to form the complex system voltage $v(k)$ (known as Clarke's transformation [25]), given by

$$v(k) = v_\alpha(k) + \jmath v_\beta(k) = Ae^{j(\omega k+\phi)} \tag{8}$$

Fig. 2(a) illustrates that for a balanced system state, the probability density function of $v(k)$ is rotation invariant (circular), since v and $ve^{j\theta}$ have the same distribution for any real θ. Statistically, this means that $v(k)$ is second order circular (proper) and with equal powers in v_α and v_β, and thus the covariance matrix, $\mathbf{C} = E[\mathbf{vv}^H]$, is sufficient to fully describe the second order statistics, while the pseudocovariance matrix, $\mathbf{P} = E[\mathbf{vv}^T] = \mathbf{0}$, vanishes as discussed in Section 2. However, when the three-phase power system deviates from its nominal condition, such as when the three channel voltages exhibit different levels of dips or transients, V_a, V_b, V_c are not identical and the complex $\alpha\beta$ voltage becomes