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A linearized approach to the Symmetric Fuzzy Power Flow for the application to real systems

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ABSTRACT

Many applications of Fuzzy Power Flow have been proposed not only for operational purposes considering uncertainties, but also for planning exercises with high level of intermittent sources, interconnection presence and, more recently, electric vehicles load. However, their use in real systems is not usual, mainly where the uncertainty level can be significant. This is due to the low accuracy of the results related to the classical methods, and the computational burden needed to achieve a high level of accuracy in the symmetric approaches. This paper aims to present a linearization of the Symmetric Fuzzy Power Flow in order to reduce the computational effort and make it possible for it to achieve high levels of accuracy when applied to real systems. With the purposes of demonstrating the applicability of the proposed approach, several IEEE test systems and a planning configuration of the Portuguese Transmission System will be studied.

1. Introduction

Uncertain variables are usual in power systems and they should be considered in planning and forecasting problems [1]. In fact, a more complex analysis, such as load flow, cannot be performed only taking into account a deterministic approach. Therefore, tools capable of modeling uncertainty are needed for power system evaluation. Borkowska presented the concept of the Probabilistic Power Flow (PPF), applied to the DC model, in which uncertainties in nodal power injections are modeled by probabilistic functions [2]. Later, this analytical version of PPF was developed for the AC formulation [3] and, afterwards, numerical approaches [4] were also applied. In the context of analytical PPF, the Boundary Load Flow (BLF) [5] was developed. The main objective of this algorithm is to evaluate the maximum and minimum values that the state variables of Power Flow (PF) problem can assume.

Nevertheless, the majority of uncertain variables in power systems can be viewed also as non-probabilistic, mainly under operational conditions. The magnitude of load in a certain moment depends, for instance, on economic growth, social conditions and human behavior; these are always changing. Therefore, in some cases, using past experiences to evaluate events in the future can be an incomplete approach. On the contrary, the uncertainties in real power systems are usually related with imprecise qualitative information. Furthermore, they can be also linked with some aspects of common language: sentences such as "generation between 15 and 25 MW" and "load around 5 MW" are typical examples of vague information that come from the experience of the System Operator.

The non-probabilistic nature of the power system inputs under operational conditions lead to alternative ways of modeling uncertainty. Imprecise data is usually modeled using fuzzy sets [6–8], which can reproduce and analyze the vagueness and inexactitude presented in some variables in terms of mathematical precision. In order to consider fuzzy uncertainties in load flow studies, Miranda and Matos [9] proposed the Fuzzy Power Flow (FPF). This approach aims to calculate the unknowns of Power Flow problem, considering that the power injections in the nodes are described using possibility distributions, modeled by fuzzy membership functions [7,8].

The introduction of possibility models in load flow analysis was widely accepted. In [10,11], arithmetic interval are used to solve the uncertain load flow problem. Although it is a particular case of fuzzy models, it was presented as an alternative way of representing the boundaries of the uncertainty. It paved the way to a new BLF approach [12], which takes imprecise and vague information as input variables rather than probabilistic density functions.

Since its first presentation, FPF has been applied to the operation and planning of power systems, either in generation transmission [13,14] or in distribution systems [15,16], leading to new

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requirements and approaches for FPF. Therefore, throughout the years, some changes were implemented to the classic formulation of FPF. For instance, in [17], some enhancements were introduced in order to consider correlated data between the nodes.

This paper will describe some of those improvements prior to the development of the Symmetric Fuzzy Power Flow (SFPF). Afterwards, a methodology to reduce the computational time of SFPF will be presented, so that the application in large-scale systems can be possible.

2. Fuzzy Power Flow: Classic and revisited approaches

Over the years, several versions of the FPF have been proposed. In general, they are divided into different applications from radial distribution systems to massed transmission systems. In terms of methodologies, they can be viewed as a classic approach and a revisited FPF approach.

2.1. Classic approach

The classic approach of FPF, proposed in [9] and developed in [18], relies on the application of the fuzzy sets theory to the traditional PF equations, both in the AC and DC models.

The DC formulation algorithm starts by running a deterministic DC PF, in order to calculate the crisp values of the bus angles (θ_d) and the line flows (P_{dik}). Afterwards, the possibility distributions that describe the active power at the nodes are specified. These possibility functions are calculated based on the crisp value as well as on the given uncertainty deviations (e.g. 5%). Normally, Triangular Fuzzy Numbers and Trapezoidal Fuzzy Numbers [7] are used to represent these distributions. The power injection uncertainty at each bus corresponds to the deviations ($\Delta\tilde{P}$) between the extremes of the triangle/trapezoid and the deterministic value. Considering the admittance matrix [B] of the deterministic DC model and the sensitivity matrix [A], the angles and flows deviations can be calculated as follows:

$$[\Delta\tilde{\theta}] = [B]^{-1} \cdot [\Delta\tilde{P}] \tag{1}$$

$$[\Delta\tilde{P}_{ik}] = [A] \cdot [\Delta\tilde{P}] \tag{2}$$

The membership functions of the resulting angles ($\tilde{\theta}$) and lines ($\tilde{P}_{ik}$) can be calculated by superimposing the deterministic values and the deviations:

$$[\tilde{\theta}] = [\theta_d] + [\Delta\tilde{\theta}] \tag{3}$$

$$[\tilde{P}_{ik}] = [P_{dik}] + [\Delta\tilde{P}_{ik}] \tag{4}$$

The AC FPF extends the DC formulation to the uncertainties of the reactive power, voltage magnitudes and losses. Therefore, the AC formulation aims to determine the possibility distributions of voltage magnitudes ($\tilde{P}$) and phases ($\tilde{\theta}$) at the nodes as well as currents and losses in the lines, considering a given range of power injections, which are represented by fuzzy numbers.

Similarly to the DC case, the classic algorithm for AC FPF starts by running a deterministic PF, adopting, for instance, the Newton–Raphson method, so that crisp values of the voltages (V_d) and angles ($\tilde{\theta}_d$) can be obtained. Afterwards, the power deviations are determined for all buses, excluding the slack bus. Using the inverted Jacobean matrix that results from the deterministic PF, it is possible to calculate the voltage and angle deviations and, consequently, the respective membership functions:

$$\begin{bmatrix} \Delta\tilde{\theta} \\ \Delta\tilde{V} \end{bmatrix} = [J]^{-1} \begin{bmatrix} \Delta\tilde{P} \\ \Delta\tilde{Q} \end{bmatrix} \tag{5}$$

$$[\tilde{\theta}] = [\theta_d] + [\Delta\tilde{\theta}] \tag{6}$$

$$[\tilde{V}] = [V_d] + [\Delta\tilde{V}] \tag{7}$$

The fuzzy values regarding active and reactive flows can be obtained using the same superposition principle. Therefore, the power deviations related to the branch i–k ($\Delta\tilde{P}_{ik}$ and $\Delta\tilde{Q}_{ik}$) are calculated through the linearization of the flows equations, using the first term of their expansion in the *Taylor Series* around the *crisp values* obtained in the deterministic PF (V_{d_i}, V_{d_k}, θ_{d_i} and θ_{d_k}):

$$\Delta\tilde{P}_{ik} = \frac{\partial P_{ik}}{\partial V_i}\Delta\tilde{V}_i + \frac{\partial P_{ik}}{\partial \theta_i}\Delta\tilde{\theta}_i + \frac{\partial P_{ik}}{\partial V_k}\Delta\tilde{V}_k + \frac{\partial P_{ik}}{\partial \theta_k}\Delta\tilde{\theta}_k \tag{8}$$

$$\Delta\tilde{Q}_{ik} = \frac{\partial Q_{ik}}{\partial V_i}\Delta\tilde{V}_i + \frac{\partial Q_{ik}}{\partial \theta_i}\Delta\tilde{\theta}_i + \frac{\partial Q_{ik}}{\partial V_k}\Delta\tilde{V}_k + \frac{\partial Q_{ik}}{\partial \theta_k}\Delta\tilde{\theta}_k \tag{9}$$

2.2. Motivation for changing the classic FPF approach

The classic approach previously presented was developed under the assumption that the uncertainties regarding power injection in the nodes are relatively narrow. This assumption is necessary to ensure the possibility of a linearization around the crisp values. Nevertheless, in some cases, the linearization does not give fully satisfactory results. For example, in the calculation of the branch flows, the values obtained for lightly loaded lines are usually affected by errors related with the linearization procedure.

Another imprecision in FPF results is associated with the slack bus. In fact, the membership function of this node is determined at the end of the FPF algorithm, following the same principle applied to the deterministic PF. In the crisp PF, the slack bus represents a mathematical artifice that is used to solve the Newton–Raphson method and to compensate for the non-predictable losses that come from the resulting flows. However, in classic FPF, the slack bus also compensates the uncertainty associated with other nodes.

In order to understand this phenomenon, one can consider a system with 3 nodes and 3 lines, as presented in Fig. 1a. Just for simplification reasons, a DC model with equal branches is assumed and bus 1 was chosen as the slack. If the maximum demand happens in bus 3 (4 MW) and node 2 is generating at the minimum level (1 MW), bus 1 should generate 3 MW. On the other hand, if bus 3 is consuming at a low level (2 MW) and the maximum

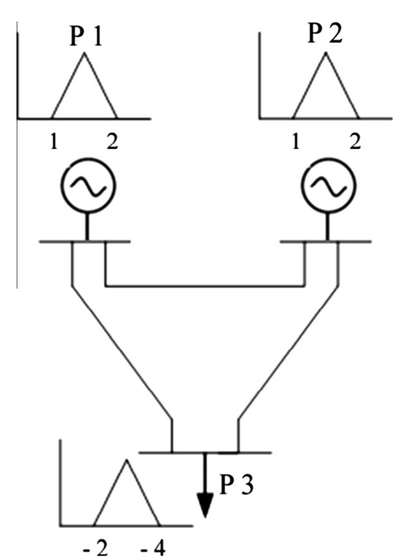

Fig. 1a. Assumed uncertainties.