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The numerical Laplace transform: An accurate technique for analyzing electromagnetic transients on power system devices

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ABSTRACT

A detailed description of the numerical Laplace transform (NLT) for electromagnetic transient calculation on power system devices under linear and non-linear conditions is presented in this paper. The development and main advantages of the NLT are reviewed, as compared to the conventional time domain simulation, including current practices for reducing numerical errors derived from data truncation and discretization of the analytical equations. A simple technique based on the superposition principle to include non-linear conditions in the frequency domain is also fully described. Besides, important results obtained recently with the NLT for different power components are presented, including comparisons with widely used time domain methods, such as the method of characteristics, and the professional simulation program EMTDC. Such comparisons reveal a high accuracy of the numerical Laplace transform when applied to the presented studies.

1. Introduction

Switching operations, faults and lightning can produce severe transient overvoltages that are dangerous to power system components such as transmission lines, underground cables, rotatory plant, transformers, grounding systems, etc. An accurate analysis of these disturbances is very important for insulation coordination design and testing stages, and is usually performed either with time or frequency domain methods. In general the former are preferred, mainly due to their ability to include non-linear and time varying elements, the short computer times required, and the possibility to perform real time simulations. In addition, the time domain program EMTP (Electromagnetic Transients Program), is nowadays the most widely used tool for simulation of electromagnetic transients in power systems [1].

As an inherent nature of electromagnetic transients models of power system devices, their electrical parameters are frequency dependent. The inclusion of this dependence is difficult in time domain models, especially for highly complex geometrical configurations of transmission lines and underground cables. Several approaches have been applied to overcome this problem since early 70s [2–7], but even the most advanced line and cable models consider approximations that are prone to errors in highly frequency dependent systems [8]. In contrast, when using frequency domain methods, such as those based on the Fourier or Laplace transforms [9–17,30], frequency dependent elements are included in a straightforward manner, since these parameters can be analytically described in the frequency domain. Thus, a method based in this domain offers the most theoretically exact transient solution.

Frequency domain methods are in general linear and time invariant, which precludes the analysis of switched networks and the inclusion of non-linear components. However, the superposition principle has been applied to overcome this situation with successful results [8,17,18]. The numerical Laplace transform has been applied to analyze transients in particular elements such as uniform transmission lines, as well as nonuniform and field excited transmission lines, underground cables, transformer and machine windings, etc. [8,18–23]. Besides, the NLT has been widely used in testing new time domain model developments.

In this article the basic development of the numerical Laplace transform is described, discussing fundamental sources of numerical errors and current practices to reducing them. Besides, a simple technique to take into account non-linear conditions in simulations is included. Significant results obtained with this technique in the field of power system transients are presented, considering 3 different application cases:

(a) Sequential energization of transmission line.
(b) Fast transient overvoltage in machine winding.
(c) Transient response of an underground cable transmission system.

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All these examples consider frequency dependent effects. Results are compared with time domain simulations performed with the professional program EMTDC and the method of characteristics.

2. Basic development of the NLT

Let $f(t)$ be a causal time domain function and $F(s)$ its image in the frequency domain. Defining the Laplace variable as $s = c + j\omega$, direct and inverse Laplace transforms are given by

$$F(c + j\omega) = \int_0^{\infty} [f(t)e^{-ct}]e^{-j\omega t}dt \tag{1a}$$

$$f(t) = \frac{e^{ct}}{2\pi}\int_{-\infty}^{\infty} F(c + j\omega)e^{j\omega t}d\omega \tag{1b}$$

where ω is the angular frequency and c is a stability constant. It can be noticed that when $c = 0$, (2a) and (2b) correspond to the Fourier transforms

$$F(j\omega) = \int_0^{\infty} f(t)e^{-j\omega t}dt \tag{2a}$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega \tag{2b}$$

Comparison of (1a) and (2a) shows that the Laplace transform can be obtained applying the Fourier integral to $f(t)\exp(-ct)$, i.e., a damped version of $f(t)$. Hence, c is also known as damping constant and, as will be seen, its correct definition is fundamental to reduce aliasing errors.

The application of (1a) and (1b) (or (2a) and (2b)) for real practical systems can be very difficult or even impossible. In consequence, these expressions have to be evaluated numerically, giving rise to truncation and discretization errors. Practical techniques for reducing numerical errors when inverting from Laplace frequency domain to time domain are addressed in the following subsections.

2.1. Truncation errors

Assuming in this Section that $c = 0$, (2b) is numerically evaluated in the finite range $[-\Omega, \Omega]$ as follows

$$f(t) = \frac{1}{2\pi}\int_{-\Omega}^{\Omega} F(j\omega)e^{j\omega t}d\omega \tag{3}$$

where Ω is the maximum frequency. Eq. (3) can be rewritten as

$$f'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)H(\omega)e^{j\omega t}d\omega, \tag{4}$$

where

$$H(\omega) = \begin{cases} 1, & -\Omega \leqslant \omega \leqslant \Omega \\ 0, & \omega < -\Omega \quad \text{and} \quad \omega > \Omega \end{cases} \tag{5}$$

From (2b) and (4)

$$F'(j\omega) = F(j\omega)H(\omega) \tag{6}$$

and from the convolution theorem

$$f'(t) = f(t) * h(t), \tag{7}$$

where $h(t)$ is the inverse Laplace transform of $H(\omega)$, computed as follows

$$h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{j\omega t}d\omega = \frac{\Omega}{\pi}\frac{\sin(\Omega t)}{\Omega t} \tag{8}$$

According to (7) and (8), truncation of the frequency spectrum is equivalent to the convolution of $f(t)$ and a *sinc* function in time domain. As an example, let $f(t)$ be a unit step function (Fig. 1b).

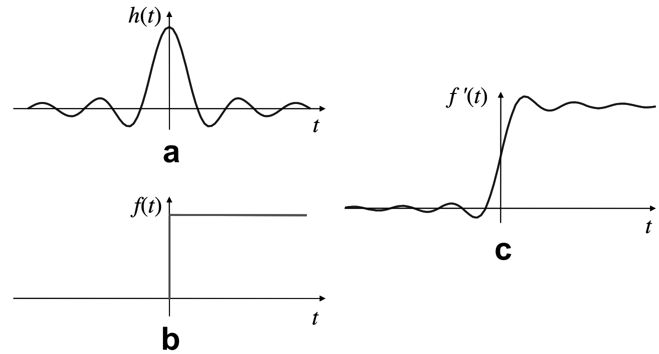

Fig. 1. Convolution of $f(t)$ and $h(t)$.

Waveform obtained from its convolution with $h(t)$ (Fig. 1a) shows high frequency oscillations near the discontinuities (Fig. 1c), known as Gibbs oscillations, which lead to amplitude errors unacceptable for transient analysis purposes. This magnitude can be reduced to an acceptable value by the introduction of some suitable data window $\sigma(\omega)$, e.g., multiplying $F(j\omega)$ by $\sigma(\omega)$. Among a variety of existing data windows for digital signal processing, Day et al. introduced in 1965 the use of the Lanczos window for transient analysis [10], while Wedepohl proposed in 1983 the use of the Hamming window [24]. More recently, the Hanning (Von Hann) and Blackman windows have also been tested, yielding satisfactory results [8]. Fig. 2 shows these data windows, while Table 1 lists their respective equations. It should be noted that these equations are valid for $|\omega| < \Omega$.

2.2. Discretization errors

Eq. (2b) can be expressed in discrete form as

$$f_1(t) = \frac{\Delta\omega}{2\pi}\sum_{n=-\infty}^{\infty} F(jn\Delta\omega)e^{jn\Delta\omega t} \tag{9}$$

where $\Delta\omega$ is the spectrum integration step. From the sampling property of a Dirac function, the term inside the summation can be expressed as follows

$$f_1(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)G(\omega)e^{j\omega t}d\omega, \tag{10}$$

being $G(\omega)$ a Dirac comb in the frequency domain

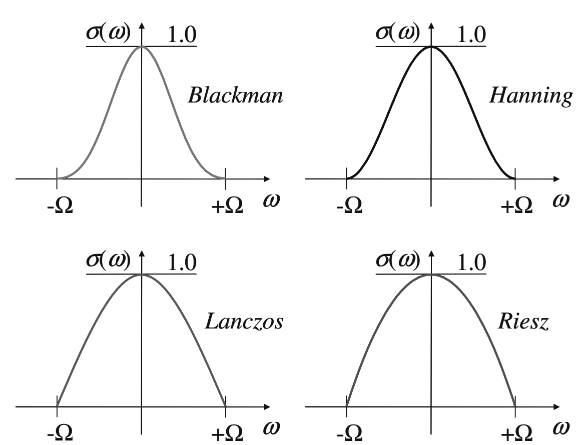

Fig. 2. Data windows.