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Semidefinite programming for optimal power flow problems ☆

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Abstract

This paper presents a new solution using the semidefinite programming (SDP) technique to solve the optimal power flow problems (OPF). The proposed method involves reformulating the OPF problems into a SDP model and developing an algorithm of interior point method (IPM) for SDP. That is said, OPF in a nonlinear programming (NP) model, which is a nonconvex problem, has been accurately transformed into a SDP model which is a convex problem. Based on SDP, the OPF problem can be solved by primal–dual interior point algorithms which possess superlinear convergence. The proposed method has been tested with four kinds of objective functions of OPF. Extensive numerical simulations on test systems with sizes ranging from 4 to 300 buses have shown that this method is promising for OPF problems due to its robustness.

Keywords: Optimal power flow; Interior point method; Semidefinite programming

1. Introduction

Since the early 60s of the last century, optimal power flow (OPF) as a powerful tool for power system optimization problems has attracted many researchers over the world [1,2]. Numerous algorithms have been developed in this area based on linear programming (LP), quadratic programming, Newton's method, nonlinear programming (NLP) and decomposition method [3]. Recently, NLP-based algorithms using interior point methods (IPM) have also been applied to OPF problems successfully [4–7]. However, as power systems are getting more complex, the OPF problems turn to be more difficult to handle. Although the theory on IPM for NLP has been well developed, many issues remain open when building the links between the modeling and the associated algorithms. First of all, the NLP-based OPF has the convergence problem due to its nonconvex nature. Moreover, in order to use IPM for NLP, the Jacobian matrices (the first-order partial derivatives) and the Hessian matrices (the second-order partial derivatives) have to be derived for each specific problem. As a result, it is not convenient to develop a general and uniform software solution for the NLP problems using IPM.

The semidefinite programming (SDP) [8,9] has been one of the most active fields in numerical optimization for over a decade. Many well-known algorithms with uniform frameworks have been exploited [10]. It has been proven that the SDP is convex and the primal–dual interior point algorithms for SDP may possess superlinear convergence theoretically [11]. Moreover, the major advantage for the SDP-based IPM is the avoidance of deriving and computing the Jacobian matrices and the Hessian matrices for each particular problem.

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Different practical problems have been solved successfully in various domains, such as control and system theory [12,13], signal processing and communications [14–16], and combinatorial optimization [17–19]. In [20,21], the SDP has been used to solve power dispatch problems, which are typical constrained economic dispatching problems (CED). To our knowledge, these two paper were pioneering in applying the SDP technique to power systems. However, the works in [20,21] did not involve power flow equations and the bus voltage constraints.

It is significant and challenging to extend the SDP to solve OPF problems which are know for their inherent complexity and practicality. Motivations of this study stem from the following three aspects:

1. The classical OPF problem is a nonconvex NLP [22], which solution is more complicated than the CED problems mentioned earlier. On the other hand, the SDP belongs to convex optimization [23], and can guarantee global optimal solution using IPM. Therefore, it is worth to study how to properly reformulate the classical OPF to a SDP model, due to many advantages of the SDP technique including its convexity and uniform algorithm implemental framework of software suite.
2. Once an OPF problem is reformulated to a SDP one using the quadratic model [24] in the case of the rectangular form, the resulting SDP model should be convex, and the solution quality can be guaranteed by using IPM for the SDP [23]. In detail, by applying $X = x^{\mathrm{T}}x$ where x is a row vector and trace operator of matrices which will be discussed in Section 3, the nonlinear quadratic items of the OPF will be replaced with the relevant elements of the variable matrix X in SDP.
3. Mature techniques such as IPM for LP are available to solve the SDP problems. Furthermore, the solution techniques developed for LP which actually is a special case of SDP are also applicable to SDP [23]. Therefore, IPM for LP, which can be used to solve convex optimization problems in polynomial time, can be implemented for SDP [25]. Moreover, IPM enhancements for LP can also be used in SDP, which renders IPM for SDP as efficient as that for LP theoretically [26].

2. Formulation of OPF problem

The OPF problem is a large-scale nonlinear optimization problem. It can be formulated in polar, rectangular, or mixture of polar and rectangular forms. In this study, the rectangular version of the OPF problem is adopted to take the advantages that the power flow equations are quadratic polynomials without trigonometric functions, which can then yield the SDP models straightforward. The objective functions to be minimized are chosen out of active or reactive power loss of transmission lines, fuel cost, and total system active power loss. Therefore, the OPF problem can be formulated as follows:

$$\text{minimize}\begin{cases} P_{\text{Loss2}} = -\sum_{i\in S_B}\sum_{j\in S_B} G_{ij}[(f_i-f_j)^2+(e_i-e_j)^2] \\ Q_{\text{Loss}} = \sum_{i\in S_B}\sum_{j\in S_B} B_{ij}[(f_i-f_j)^2+(e_i-e_j)^2] \\ F_{\text{Cost}} = \sum_{i\in S_G}(a_{fi}+a_{li}P_{Gi}+a_{qi}P_{Gi}^2) \\ P_{\text{Loss1}} = \sum_{i\in S_G} P_{Gi} \end{cases} \tag{1}$$

subject to:

1. Power flow equations:

$$\begin{cases} P_{Gi} - \sum_{j\in S_B}[e_i(e_jG_{ij}-f_jB_{ij})+f_i(f_jG_{ij}+e_jB_{ij})] = P_{Di} \\ Q_{Ri} - \sum_{j\in S_B}[f_i(e_jG_{ij}-f_jB_{ij})-e_i(f_jG_{ij}+e_jB_{ij})] = Q_{Di} \end{cases}, \quad i\in S_B \tag{2}$$

2. Constraint of reference bus:

$$e_s = 1.05; \quad f_s = 0 \tag{3}$$

3. Limits of active and reactive power:

$$\underline{P}_{Gi} \leqslant P_{Gi} \leqslant \overline{P}_{Gi}, \quad i\in S_G \tag{4}$$

$$\underline{Q}_{Ri} \leqslant Q_{Ri} \leqslant \overline{Q}_{Ri}, \quad i\in S_R \tag{5}$$

4. Limits of voltage at each bus:

$$\underline{V}_i^2 \leqslant (e_i^2+f_i^2) \leqslant \overline{V}_i^2, \quad i\in S_B \tag{6}$$

where

a_{fi}, a_{li}, a_{qi}: cost coefficients of thermal plant, i, respectively,
e_i, f_i: real and imaginary part of voltage at bus i,
$\dot{V}_i$: voltage at bus i,
G_{ij}, B_{ij}: real and imaginary part of transfer admittance between buses i and j,
P_{Gi}, Q_{Ri}: dispatchable active and reactive power at bus i,
P_{Di}, Q_{Di}: active and reactive power demand at bus i,
S_B, S_G, S_R: set of buses (n_B), thermal plants (n_G) and reactive power sources (n_B), respectively,
s: identification serial number of reference bus in the system,
$(\underline{\circ})$, $(\overline{\circ})$: lower and upper limits of variables or functions.

The set of (1)–(6) is known as the classical OPF problem.

3. Semidefinite programming by IPM

The semidefinite programming [9] is concerned with choosing a positive semidefinite matrix to optimize a linear function which is subject to linear constraints. In other words, the well-known linear programming problem is generalized by replacing the vector of variables with a symmetric matrix and the nonnegative constraints with a positive semidefinite constraint. This generalization nevertheless inherits several important properties from its vector