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Probability distributions of outputs of stochastic economic dispatch

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ABSTRACT

This paper introduces an approach to carry the uncertainty about the wind speed through the optimization for the economic dispatching. This approach uses simulated wind speed data points for inputs to economic dispatch models and produces data for estimating the probability distributions of optimal fossil fuel generation outputs, transmission loss, and total cost of power generation. Large samples of simulated data allow statistical analyses of the outputs such as the mean, median, percentiles, and confidence intervals for each output variable, plots of distributions of all output variables, and correlation measures for examining relationships between pairs of the output variables. Through the proposed algorithm, a generation expansion planner can perform useful analyses without executing the algorithm repeatedly. The algorithm and its results are illustrated through an example of an economic dispatch model and a model for the distribution of the wind speed based on a forecast for the planning target time.

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Introduction

In electric power system literature, an Economic Dispatch (ED) model is used to minimize the cost of production of power to be generated by several sources for a target time. With the increasing penetration of the wind energy in power systems, one of the challenges facing the system operators is how to incorporate the wind energy potential and reliably dispatch an optimal combination of all the available energy sources [1–16]. The primary problem associated with incorporating the wind power into the ED model is the uncertainty about the wind speed for the dispatching target time which is usually a day ahead. The stochastic aspect of the wind speed has been addressed in the literature through inclusion of the probability density function (PDF) of the wind power generation and the expected cost of over and/or under estimation of the wind speed into the ED model [3,4]. In addition, [5] has used Monte Carlo simulation and a forecasting model to generate short term wind speed forecast for the ED model. These stochastic ED methods provide optimal combinations of the available energy sources to be dispatched. However, thus far producing the probability distributions of the optimal solutions in the context of the stochastic ED models has not been considered. This paper fulfills this void.

Producing the probability distributions of the optimal solutions in the closely related problem of *optimal power flow* (OPF) is commonplace. The Monte Carlo simulation and some approximation methods are used for finding the solutions' PDFs in the OPF problem [17–21]. This paper proposes a Monte Carlo simulation algorithm for producing PDFs of the optimal solutions of the stochastic ED models, hereafter referred to as *S-ED*.

We will describe the proposed algorithm, illustrate it through an example, and discuss how a generation expansion planner can perform many useful analyses using the outputs of S-ED. For example the generation expansion planner can do the economic dispatch of the fossil power plants based the mean, median, percentiles of the distributions of the optimal outputs of the fossil fuel power plants, minimum operation costs, and transmission losses. Based on a range of the wind speed with a given probability, a planner can determine the corresponding ranges of the output variables. Similarly, for a sub-range of the optimal operation range of any fossil fuel power plant, the planner will be able to determine its probability as well as the corresponding sub-ranges of the optimal operation ranges of the remaining generators and their probabilities. In addition, the planner can perform "what if" analyses in order to compare the probability distributions of the optimal outputs of the fossil fuel power plants, the transmission losses, and optimal operation costs under various scenarios.

This paper is organized as follows. Section 'Stochastic inputs and outputs' describes the proposed S-ED algorithm. Section 'Illustration of S-ED algorithm' illustrates the S-ED algorithm through an

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example that includes Monte Carlo simulation from a wind speed forecast model. Section 'Planning applications' discusses a few planning applications. Section 'Conclusions' gives the concluding remarks. An Appendix A provides details on Monte Carlo simulations based on a wind speed series data and a forecast available for the planning time and defines the notion of stochastic order used for comparing the distributions of the optimal solutions.

Stochastic inputs and outputs

The wind speed for planning target time t will be denoted by $W(t)$ and its distribution will be denoted by $F_{W(t)}$; the dependence of the wind speed, its distribution, all subsequent variables on the target time t are explicitly emphasized. The wind power used in the ED models is a function of the wind speed, $P_{W(t)} = q(W(t))$, thus the random fluctuation of the wind speed induces randomness into the wind power, thereby into the ED model [22,23]. Due to the randomness of $P_{W(t)}$, all of the outputs of an ED model are subject to random fluctuations. The following relationship between the wind power at the planning time, $P_{W(t)}$ and $W(t)$ is commonly used:

$$P_{W(t)} = q(W(t)) = \begin{cases} 0 & W(t) \leqslant V_{CI} \quad \text{or} \quad W(t) \geqslant V_{CO} \\ \frac{(W(t)-V_{CI})W_r}{V_r - V_{CI}} & V_{CI} \leqslant W(t) \leqslant V_r \\ W_r & V_r \leqslant W(t) \leqslant V_{CO} \end{cases} \tag{1}$$

where V_{CI}, V_r and V_{CO} are cut-in, rated and cut-out wind speeds respectively, and W_r is the rated wind power.

Let $f_{w(t)}$ denote the probability density function (PDF) of the distribution of the wind speed $F_{w(t)}$. Then the probability distribution of the wind power random variable, $P_{w(t)}$, can be represented as follows: a continuous PDF $g(P_{w(t)})$ over the interval (0, W_r) where,

$$g(P_{W(t)}) = \frac{V_r - V_{CI}}{W_r} f_{W(t)}(P_{W(t)}), \quad 0 \leqslant P_{W(t)} \leqslant w_r \tag{2}$$

and two probability atoms at the end points of the interval $0 \leqslant P_{w(t)} \leqslant w_r$ given by

$$Pr(P_{W(t)} = 0) = 1 + F_{W(t)}(V_{CI}) - F_{W(t)}(V_{CO}), \tag{3}$$

$$Pr(P_{W(t)} = w_r) = F_{W(t)}(V_{CO}) - F_{W(t)}(V_{CI}). \tag{4}$$

The outputs of an ED model with N fossil fuel generators, include a vector of optimal powers, $\boldsymbol{P}_{\text{ED}} = (P_{1,\text{ED}}, \ldots, P_{N,\text{ED}})$, the total operation cost OC, and the total transmission loss P_L. The time-variant random input, $P_{W(t)}$, of the ED model produces time-variant random fossil fuel power vector, $\boldsymbol{P}_{\text{ED}}(t) = (P_{\text{ED},1}(t), \ldots, P_{\text{ED},N}(t))$.

The inputs of the proposed S-ED algorithm are Monte Carlo samples $W^1(t), \ldots, W^M(t)$ from a wind speed distribution. For each simulated wind speed, $W^h(t)$, $h = 1, \ldots, M$, an of outcome is computed using Eq. (1), which provide a random sample of size M from the probability distribution of the wind power Eqs. (2)–(4).

For each simulated input of the wind power, $P^h_{W(t)} = g(W^h(t))$, $h = 1, \ldots, M$, the ED model uses the power demand, fuel cost coefficients, transmission loss coefficients, and the system constraints as inputs. The ED model produces, $\boldsymbol{P}^h(t) = (P_1(t), \ldots, P_N(t))^h$, $OC^h(t)$, and $P^h_L(t)$. Upon using the entire set of simulated samples of the wind power, $P^h_{W(t)} = g(W^h(t))$, $h = 1, \ldots, M$, the ED model provides Monte Carlo samples of size M from the probability distributions of the optimal random vector $\boldsymbol{P}_{\text{ED}}(t)$, the operation cost $OC_{\text{ED}}(t)$, and the transmission loss. These samples are made available through simulations where mathematical functional relationships between the inputs and the outputs of an ED model are not available in closed-forms.

The Monte Carlo samples can be used to estimate distributional functions such as PDFs, cumulative probability distributions, reliability functions, and summary measures such the mean, standard deviation, median, percentiles, prediction intervals, correlation coefficients, and scatter plots. The algorithm is capable of generating a large number of samples, M, such that by the Law of Large Numbers, the distributions of, $\boldsymbol{P}_{\text{ED}}(t) = (P_{\text{ED},1}(t), \ldots, P_{\text{ED},N}(t))$, $OC_{\text{ED}}(t)$, and $P_{\text{ED},L}(t)$ can be reliably estimated. The estimates of the entire distributions of the optimal outputs allow distributional comparison in their entirety such as stochastic ordering [24] defined in Appendix A and illustrated in the next section.

Illustration of S-ED algorithm

This section illustrates the implementation and results of the S-ED algorithm for a wind-penetrated system consisting of one wind farm, and $N = 3$ fossil fuel power generators for $P_D = 210$ MW power demand.

Wind speed simulation model

For simulating wind speed data, we assume that a forecast, $W_f(t)$, for the target time t is available. (Developing the forecast is beyond the scope of this paper and we refer the reader to [25–30].) We assume that $W_f(t)$ is a non-stochastic function of t, which provides the most likely wind speed for the target time. That is, $W_f(t)$ is the mode of the wind speed PDF $f_{W(t)}$. This assumption corresponds to assuming that the actual wind speed of the target time will be $W_f(t)$ subject to a random error whose most likely value is zero. The following model combines the available forecast and the error:

$$W(t) = W_f(t) + m(t)\varepsilon. \tag{5}$$

where ε is a random variable with probability distribution F_ε and $m(t) > 0$ is a tuning parameter for determining the scale of the wind speed distribution $F_{W(t)}$ such that $W_f(t)$ is the mode of $f_{W(t)}$ and the mode of f_ε is at zero. Note that $m(t)$ is the ratio of the scale parameters of the wind speed and error distributions.

Model Eq. (5) implies that $f_{W(t)}$ has the same shape parameter as f_ε, but its mode and scale are different from those for f_ε. For example, in forecasting literature it is common to assume that the error distribution F_ε is normal [5]. For such models, the mean and the median of the error distribution are also zero, the wind speed distribution is normal with the mean and median equal to $W_f(t)$, and the standard deviation equal to the scale parameter, $\sigma_{W(t)} = m(t)\sigma_\varepsilon$. However, in the power literature, often it is assumed that the wind speed distribution is a Weibull [23,31–33]. For the Weibull model with mode, $W_f(t)$, the mean, median, and the mode are different, and the corresponding error distribution is a three-parameter Weibull distribution with the following PDF:

$$f(\varepsilon) = \frac{k}{c}\left(\frac{\varepsilon - \tau}{c}\right)^{k-1} e^{-\left(\frac{\varepsilon-\tau}{c}\right)^k}, \qquad \varepsilon \geqslant \tau, \quad c > 0, \quad k > 0, \tag{6}$$

where k is the shape parameter, c is the scale parameter, and τ is referred to as the threshold parameter [34].

The two-parameter Weibull PDF for the wind speed is found by Eqs. (5) and (6) with $\tau = 0$, the shape parameter k, and the scale parameter $c_{W(t)} = m(t)c$. Using the formula for the mode of the Weibull distribution relates the model parameters as follows:

$$W_f(t) = c_{W(t)} = m(t)c\left(\frac{k-1}{k}\right)^{\frac{1}{k}}, \quad k > 1. \tag{7}$$

The forecast $W_f(t)$ induces the dynamic of time into mode and scale of the wind speed distribution, hence $F_{W(t)}$ is time-variant. The error distribution does not depend on the specific forecast for time t. Model Eq. (5) enables us to simulate the wind speed of the target planning time $W^h(t)$, $h = 1, \ldots, M$ through the static distribution F_ε instead of the time-variant distribution $F_{w(t)}$.