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Power system state estimation using a direct non-iterative method

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ABSTRACT

This paper describes a new method for state estimation of a non-linear AC power system in a non-iterative manner. This method is based on the Kipnis–Shamir relinearization technique that is used to solve over-defined sets of polynomial equations. The technique transforms the equations to a higher dimensional linear space which allows the states to be solved in a non-iterative manner. Given accurate measurements, this new state estimation method provides the same results as traditional iterative state estimation methods, and the proposed method does not require an initial guess of system states nor does it have issues with convergence.

Introduction

Since its introduction in the late 1960s [1], power system state estimation has become an integral part of power system monitoring and operation. Because state estimation for alternating current (AC) power systems is a non-linear problem, traditionally state estimation has been solved using iterative methods such as the weighted least-squares with Gauss–Newton iterations [2,3]. Iterative methods have worked well for the state estimator application, but these methods require an initial guess and may run into convergence issues if the initial guess is too far away from the actual system states [4].

This paper describes a new method for solving the state estimation problem of a non-linear AC power system in a non-iterative manner when given an adequate set of measurements [5–8]. There are other methods to solve the state estimation problem [9,10] and the related power flow problem [11,12] in a non-iterative manner by using linearized measurement functions and having some way to compensate for the linearized model. In contrast, the method proposed in this paper is based on the Kipnis–Shamir relinearization technique that is used to solve over-determined systems of polynomial equations [13]. In the proposed method, the measurement equations, which are the bus voltage magnitude, line power flow, and bus power injection equations, are formulated using rectangular representation of the bus voltages. With this formulation, the non-linear measurement equations become quadratic polynomials of the voltage variables. The method then uses two transformations to change the original system into a larger system to solve for the quadratic variables in a non-iterative manner. This new method provides the same results as the weighted least-squares method when given accurate measurements, and does not require an initial guess or have issues with solution convergence.

The main contributions of this paper are:

1. a comprehensive development of the algorithm needed for this non-iterative method, as shown in Fig. 1, including techniques to reduce computation time, and
2. an assessment of this non-iterative method as compared to the conventional iterative state estimator solution.

The remainder of this paper is organized into five sections. Section 'Non-iterative state estimation method' provides a description of the non-iterative state estimation solution process. Section 'A simple example' presents a simple example to demonstrate the mechanics of the solution method. Section 'Observability requirements' discusses the observability requirements of the proposed method. Section 'Performance test results' shows the performance test results of the proposed method, both for

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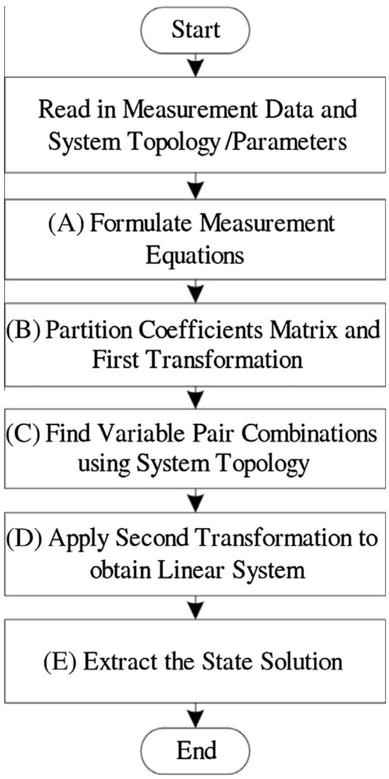

Fig. 1. Non-iterative state estimation method flow chart.

computation time and for solution accuracy when there are measurement errors. Finally, Section 'Summary and future work' provides a summary and lists some future work to be done on this research topic.

Non-iterative state estimation method

This section describes the non-iterative state estimation method. The inputs needed by the method are the system topology and parameter information and measurements from the system.

Formulating measurement equations

In the method, the measurement equations are formulated using rectangular representation of the bus voltage phasors, i.e., $\overline{V}_i = V_{iR} + jV_{iI}$, such that the equations become quadratic polynomials in terms of these real and imaginary bus voltage components. If the transmission line parameters are expressed using the π-model shown in Fig. 2, then the measurement equations are:

(a) Bus magnitude measurements

$$V_{iM}^2 = V_{iR}^2 + V_{iI}^2 \tag{1}$$

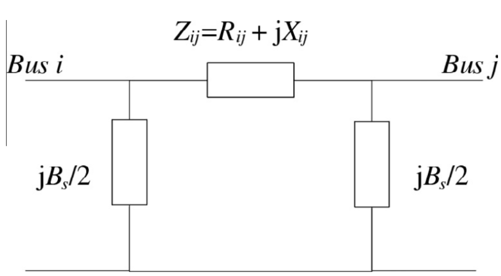

Fig. 2. Standard transmission line π-model.

where V_{iM} is the magnitude of the voltage at Bus i.

(b) Line active (P) and reactive (Q) power flow equations

$$\begin{aligned} P_{ij} &= g_{ij}(V_{iR}^2 + V_{iI}^2 - V_{iR}V_{jR} - V_{iI}V_{jI}) + h_{ij}(V_{iI}V_{jR} - V_{iR}V_{jI}) \\ Q_{ij} &= h_{ij}(V_{iR}^2 + V_{iI}^2 - V_{iR}V_{jR} - V_{iI}V_{jI}) + g_{ij}(V_{iR}V_{jI} - V_{iI}V_{jR}) \\ &\quad - \frac{B_s}{2}(V_{iR}^2 + V_{iI}^2) \\ g_{ij} &= \frac{R_{ij}}{Z_{ij}^2}, h_{ij} = \frac{X_{ij}}{Z_{ij}^2}, Z_{ij}^2 = R_{ij}^2 + X_{ij}^2 \end{aligned} \tag{2}$$

where Bus i is the from bus and Bus j is the to bus, and R_{ij}, X_{ij}, B_s are the line resistance, reactance, and shunt susceptance, respectively.

(c) Bus power injection equations are formulated by adding together all of the line flow equations that are going out of the bus plus the power flowing into any external shunt conductance G or susceptance B connected to the bus (e.g., a shunt capacitor or reactor on a bus):

$$\begin{aligned} P &= \sum P_{ij} + G(V_{iR}^2 + V_{iI}^2) \\ Q &= \sum Q_{ij} - B(V_{iR}^2 + V_{iI}^2) \end{aligned} \tag{3}$$

where j is the set of buses connected to Bus i.

Because these equations are linear with respect to the quadratic voltage terms ($V_{iR}^2, V_{iI}^2, V_{iR}V_{jR}$, etc.), the equations can be put into the matrix form

$$A_\xi \xi = C \tag{4}$$

where C is the vector of measurement values, ξ is vector of quadratic voltage variables, and A_ξ is the coefficient matrix for ξ. The vector ξ consists of quadratic variables of the real and imaginary parts of the voltages, which are denoted by $x_i x_j$, where the indices i and j are not related to the bus numbers.

Line shunt conductance, non-unity transformer tap ratios and phase shifters can be added to the transmission line model. The resulting measurement equations will be more complex, but they are still quadratic polynomials of the voltage components.

Partitioning A_ξ and first transformation

A re-ordering of variables is performed and the system (4) is rearranged into

$$\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} Y \\ Z \end{bmatrix} = C \tag{5}$$

where A contains the linearly independent columns of A_ξ, B contains the remaining columns of A_ξ, Y is the vector of elements of ξ corresponding to A, and Z is the vector of elements of ξ corresponding to B.

The quadratic variables $x_i x_j$ in Y are renamed to $y_1, y_2, \ldots, y_{N_y}$ in the order they appear, and N_y is the total number of Y variables. The quadratic variables $x_i x_j$ in Z are renamed $z_1, z_2, \ldots, z_{N_z}$ in the order they appear, and N_z is the total number of Z variables.

In addition, all quadratic variables containing the reference bus imaginary component and their corresponding columns in the matrices are eliminated from the system because the reference bus imaginary component is set to zero. In a software implementation, the partitioning process can be reliably performed by using QR decomposition [6] on A_ξ.

With the rearranged system, the Y variables can now be expressed in terms of the Z variables and the measurement values C as

$$\begin{aligned} Y &= d + DZ \\ d &= \left(A^T A\right)^{-1} A^T C, \quad D = -\left(A^T A\right)^{-1} A^T B \end{aligned} \tag{6}$$