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# Saturated robust power system stabilizers

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ABSTRACT

In this paper, a new saturated control design for uncertain power systems is proposed. The developed saturated control scheme is based on linear matrix inequality (LMI) optimization to achieve prescribed dynamic performance measures, e.g., settling time and damping ratio. In this design, the closed-loop poles are forced to lie within a desired region. The proposed design provides robustness against system uncertainties. The simulation results of both a single machine infinite bus and a multi-machine power systems are given to validate the effectiveness of the proposed controller.



## Introduction

LARGE Power system stability enhancement is of great importance, since if stability is lost, power separation and collapse may occur and negative consequences may be brought to the national economy. Generators are usually equipped with thyristor-controlled static exciter due to its rapidity, and high reliability. The terminal voltage deviation from a reference value is used to regulate the terminal voltage of generators using proportional (P) or proportional integral derivative (PID) control-termed automatic voltage regulator AVR. However, the AVR may have an adverse effect on system stability for large closed-loop gains of the excitation channel. This problem is solved by injecting an additional stabilizing signal generated by power system stabilizers (PSS) whose input is usually the speed deviation of the generator. Many PSS designs exist in the following references and references therein. A single or double lead stage control using frequency response and root locus methods are presented in [1,2]. The work in [3] provides coordinated design of AVR-PSS. Linear optimal control is reported in [4]. Robust control to consider the uncertainty due to load variations is presented [5–8]. In [9], resilient control is proposed to cope with uncertainties due to both load variations and controller parameters errors. Further stability enhancement is achieved by making use of flexible AC transmission systems (FACTS) devices [10], presents reliable (fault-tolerant) stabilization to consider the case of failure of either PSS or FACTS controllers. Intelligent PSSs based on evolutionary techniques to enhance the system response over a wide range of operating points have been proposed, e.g. [11–13].

Industrial control systems always have limitations on the amplitudes of control inputs due to saturation characteristics of actuators. These limitations may cause serious deterioration of control performances and even destroy the stability of the systems. Hence, the topic of designing control systems that maintain stability and desired performance in the presence of the saturation constraint is a topic of utmost practical interest, e.g. the excitation control of power systems [14,15]. Moreover, these plants are also subject to uncertainties due to parameter variations or un-modeled dynamics. None of the above mentioned references tackles the design of PSS taking into consideration the limitation on its output control signal. However, in [16] a procedure is developed to estimate the stability region of PSS subject to actuator saturation. A multi-objective optimization model is presented to estimate the practical stability region for a small-signal power system dynamic model with saturation nonlinearities [17]. The saturated excitation control of multi-machine multi-load power systems using a Hamiltonian function approach is presented in [18].

Many control design approaches dealing with saturating control are available in the literature, e.g. [19,20] and the references therein. One of these approaches is the positive invariance. This

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technique is based on the design of controllers that work inside a region of linear behavior where saturation does not occur. In other words, the positive invariance approach is based on constraint avoidance to prevent the saturation in the closed-loop system and as a result a region of linear behavior of the system is maintained [21–23]. Another design approach, however, allows saturation to take place and asymptotic stability is guaranteed as well [24–29]. Several works extend the last approach to deal with different kind of problems encountered in the literature as the magnitude and rate constraints [30,31]. Other designs of saturated control such as $L_1$ optimization, the small and high gain, and model predictive control have been reported in [32–34].

One of the most important problems in control design is the robust asymptotic stability (i.e. asymptotic stability against all admissible uncertainties). Several techniques are available for linear systems affected by time-invariant uncertainty. Without considering the controller's saturation, most of these methods cast the uncertainty in polytopic [35,36] or in norm-bounded forms [37] and provide LMI-based sufficient conditions for robust stability. To achieve a prescribed dynamic behavior in the presence of system uncertainties and saturation of the control signal, robust pole assignment with saturated control is proposed in [38,39]. In such approach, the LMI regional pole assignment is connected to positive invariance methods to synthesize stabilizing state feedback controllers ensuring regions of desired closed-loop poles together while avoiding the saturation. The positive invariance technique is used in this work because it provides simple methods to calculate stabilizing feedback controllers and it can deal with non-symmetrical constraints as well.

In this paper, a new approach is presented to design a saturating PSS for uncertain systems. In this new approach, the control signal is allowed to saturate while guaranteeing asymptotic stability to a bounded, ellipsoidal and symmetric region obtained by the solution of a set of LMIs. The main challenge in this approach is to obtain a large enough domain of initial states that ensures asymptotic stability for the system despite the presence of saturations. To get around this problem, linearization of the nonlinear saturation function is introduced.

The objective of the present work is to design a state feedback robust regional pole placement controller that copes with control signal saturation constraints and system uncertainties. In order to design the aforementioned controller, a convex optimization problem is formulated using the LMI method. The rest of the paper is organized as follows; Mathematical notations and problem formulation are given in Section 'Mathematical notations and problem formulation'. The problem solution is presented in Section 'Problem solution', while Section 'Saturated PSS with regional pole placement' presents simulation results of a single machine infinite bus system (SMIB) and a multi-machine system. Conclusion is given in Section 'Conclusion'.

## Mathematical notations and problem formulation

The $R^n$ and $R^{n\times m}$ denote, respectively, the $n$-dimensional Euclidean space and the set of $n \times m$ real matrices. In the sequel, $W'$, $W^{-1}$, and $\|W\|$ denote respectively the transpose, the inverse, and the induced norm of any square matrix $W$. The notation $W > 0$, $W < 0$ is used to denote a symmetric positive (negative) definite matrix $W$; $I$ denotes the identity matrix of appropriate dimension. The symbol · is as an ellipsis for terms in matrix expressions that are induced by symmetry e.g.

$$\begin{bmatrix} L + (W + N + W' + N') & N \\ N' & M \end{bmatrix} = \begin{bmatrix} L + (W + N + \bullet) & N \\ \bullet & M \end{bmatrix}$$

The following facts will be used throughout the paper [40].

**Fact 1.** *The congruence transformation z′Wz does not change the definiteness of W.*

**Fact 2.** *For any real matrices $W_1$, $\Delta(t)$, and $W_2$ of appropriate dimensions, it follows that*

$$W_1\Delta(t)W_2 + \bullet \leqslant \varepsilon W_1 W_1' + \varepsilon^{-1} W_2' W_2, \varepsilon > 0$$

*where $\Delta(t)$ represents system uncertainties with bounded norm*

$$\|\Delta(t)\| < 1 \Longleftrightarrow \Delta'\Delta < 1$$

*The usefulness of this fact is in removing uncertainty.*

**Fact 3** (*Schur complement*). *This fact is used to transform a nonlinear matrix inequality to a linear one. Given constant matrices $W_1$, $W_2$, $W_3$ where $W_1 = W'_1$, and $0 < W_2 = W'_2$. Then*

$$W_1 + W_3' W_2^{-1} W_3 < 0 \Longleftrightarrow \begin{bmatrix} W_1 & \bullet \\ W_3 & -W_2 \end{bmatrix} < 0$$

*Now the problem in hand can be formulated as follows. Consider the following uncertain system:*

$$\dot{x} = (A + \Delta A)x + Bu, x(0) = x_o \quad (1)$$

*where $A \in R^{n\times n}$ and $B \in R^{n\times m}$ are known real constant matrices that describe the nominal system. The matrix $\Delta A$ is real; time varying matrix functions representing the norm bounded parameter uncertainties and is given by:*

$$\Delta A(t) = M\Delta(t)N, \|\Delta(t)\| < 1$$

*where M, and N are known real constant matrices, with $\Delta(t)$ being an unknown, time-varying matrix function. It is worth mentioning that $\Delta(t)$ can represent system uncertainties, unmodeled dynamics, and/or nonlinearities. The pair $(A, B)$ is assumed to be controllable. The saturation control, Fig. 1, is assumed to be of the state feedback form, symmetry and normalized as defined:*

$$u = Fx, \quad -1 < u_j < +1, \quad j = 1 \dots m$$

*or*

$$sat(u_j) = \begin{cases} 1, \ if \ u_j \geqslant 1 \\ u_j, \ if \ -1 < u_j < 1, \ j = 1 \dots m \\ -1, \ if \ u_j \leqslant -1 \end{cases} \quad (2)$$

*The problem can be stated as follows: design a controller in the form of (2) for the system given in (1). The saturation controller to be designed is the one that achieves prescribed dynamic performance by forcing the closed-loop poles to lie in a specified region. To achieve a desired dynamic performance, specified by a minimum damping ratio $\zeta_{min}$, and a maximum settling time $T_s$ (or equivalently $\sigma = 4/T_s$), the closed-loop poles should lie within the disc $D(q,r)$ with center $(-q, 0)$ and radius r as shown in Fig. 2. This is termed as D-stability in which the poles must lie inside d for all admissible uncertainties. Note that $\sigma$ is called the degree of stability, or relative stability. The pole placement in a circular region is an adequate practical constraint to achieve good transient response for uncertain systems.*

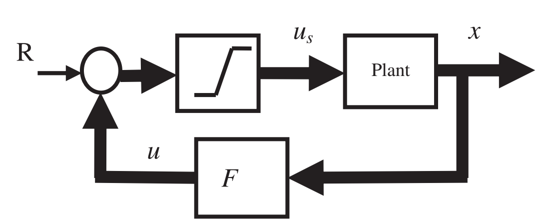


**Fig. 1.** Feedback system with saturated control.