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Stability region and radius in electric power systems under sustained random perturbations

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ABSTRACT

Two concepts are proposed to characterize the behavior of stochastic systems under sustained random perturbations in time: Using Lyapunov exponents we define the region where an electric power system can be operated under random perturbations without losing stability; and we characterize the maximum perturbation size that a system can sustain. The proposed methodology is applied to international test systems of nine and thirty-nine buses.

Introduction

In their daily operation, electric power systems are subjected to a variety of random perturbations sustained in time, due to the dynamic behavior of consumption, temperature changes in the wires, errors in the measuring instruments, changes in the network's topology, etc. Therefore, the randomness is present at all times, and it is necessary to represent it as faithfully as possible to capture the stochastic behavior of real systems.

Traditionally, there have been attempts from probabilistic theory to analyze the stochastic dynamics of electric systems, orienting the study to the analysis of contingencies and safety, see [1], where the objective consists in assigning an occurrence probability to a set of predefined events. Then, the probability that the system will be stable is estimated from the probability distributions of the elements that represent the random behavior.

In the context of dynamic stability, Refs. [2–4] analyze small signal stability, assigning a probability value to the occurrence of certain events. In [5] it is considered that consumption varies permanently in time, and an index is presented that allows the determination of the vulnerability of a system in studies of voltage collapse from the time at which the system abandons the stability region.

With respect to the probabilistic analysis of stability of small perturbations, Refs. [6–12] show important advances in this area, but the random effect is considered according to a stepwise type of event, and the sustained variation in time is not considered. To account for the above, Ref. [13] shows a theoretical development based on Lyapunov exponents that allow the characterization of the random phenomenon in electric power systems. However, no numerical methods are presented for implementation in real systems. In [14] numerical methods are reported to evaluate stability in mechanical systems by means of Lyapunov exponents, but the results shown cannot be extrapolated to large systems such as electric power systems.

In the context of the model of random variations sustained in time, white noise or Brownian motion, see [15], has been used to represent the stochastic dynamics of electric systems. However, this process is adequate for applications at the microscopic level, and it is not a correct approximation to represent the macroscopic phenomena existing in electric networks.

The present paper models the random perturbations sustained in time which affect electric power systems, according to a particular stochastic process reported in [16]. It also proposes to use Lyapunov exponents and the gains of the PSS controllers, to characterize the stability of electric systems, defining the stability region and stability radius of a system subjected to random perturbations sustained in time.

The proposed methodology is applied to two IEEE test systems: the three generator – nine bus and the ten generator – thirty-nine bus systems. The rest of the paper is organized as follows: Section 'Literature review' presents the mathematical model of linear stochastic systems and the concept of Lyapunov exponents. Sec-

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tion 'Model of the system' introduces two indicators in order to characterize random perturbations in power system operation and presents a methodology for numerical estimation. Finally, in Section 'Methods' the proposed methodology is applied to two examples of multimachine power systems, highlighting the potential applications of the presented concepts.

Literature review

Different authors have made valuable proposals that allow analyzing the small signal stability of an electric power system subjected to random perturbations self-sustained over time, [13,14] and others. However, the fact is that the reported novel and important methods are not directly applicable to international testing systems, mainly because of the large number of dynamic variables that represent the systems, and therefore the simulation of those techniques presents numerical disadvantages. The work reported in [15] shows important advances in this aspect, but the application has been focused on the analysis of the stability of mechanical structures.

Ref. [18] shows a method for tuning controlling parameters in very large electric power systems, considering a stochastic approach. The main objective of this work is to evaluate the system's response from the definition of performance indicators, considering that the perturbation that affects the system's dynamics is represented by means of an additive model self-sustained over time. The purpose is to evaluate the impact of the gains of the controllers of the machines on the cost of the energy losses under permanent regime, and in this way determine a better fit of the parameters when required.

The work reported in [19] shows the results of analyzing the small signal stability of electric systems subjected to multiplicative stochastic perturbations through the calculation of Lyapunov exponents. Three numerical methods are shown that allow determining a single Lyapunov exponent that allows generalizing the analysis of classical deterministic eigenvalues.

Ref. [23] uses the Lyapunov exponent to define stability radii in electric systems subjected to random perturbations self-sustained over time, using the numerical methods reported in [19]. This work shows a methodology that allows determining the maximum perturbation size that a system can resist without losing stability. However, the analysis is made on a test system that considers a generator connected to an infinite busbar.

Ref. [26] uses the methods reported in [19] to define performance indicators in linear stochastic systems subjected to random perturbations that are represented by a multiplicative model.

The present paper follows the guidelines of previous papers. A method is shown that allows determining stability radii and regions in multimachine electric systems subjected to random perturbations self-sustained over time. The perturbations are represented by means of a multiplicative model in which the stability radii and regions are determined from the calculation of Lyapunov exponents.

Model of the system

Basic concepts

A system of linear differential equations, with constant coefficient matrix, can be written in the form

$$\Delta\dot{x} = A\Delta x \quad \text{in } \mathbb{R}^d. \tag{1}$$

To analyze the stability of the linear system (1) it is necessary to determine the real parts of the eigenvalues of the matrix A. The system will be asymptotically and exponentially stable if and only if all the real parts of the eigenvalues are negative. However, this result is not valid for systems that vary in time as follows (see [20])

$$\Delta\dot{x} = A(t)\Delta x \quad \text{in } \mathbb{R}^d. \tag{2}$$

In this context, it becomes necessary to consider a different approach to stability studies, and the theory of Lyapunov exponents allows this problem to be solved.

Let us consider a linear system in which the variation is stochastic and is sustained in time

$$\Delta\dot{x} = A(\xi_t)\Delta x \quad \text{in } \mathbb{R}^d, \tag{3}$$

where ξ_t represents the random and time-varying effect, by means of a Markov-type stochastic process. If we denote the solution of (3), for an initial condition $x_0 \in \mathbb{R}^d$, by $\varphi(t, x_0, \xi_t)$, then the exponential growth behavior of the linear system is given by the Lyapunov exponents

$$\lambda(x_0, \omega) = \limsup_{t\to\infty} \frac{1}{t} \log \|\varphi(t, x_0, \xi_t(\omega))\|. \tag{4}$$

In this case, ω is an element of the probability space on which the differential stochastic Eq. (3) is defined. Note that the trajectory $\varphi(t, x_0, \xi_t(\omega))$ is (exponentially) stable if and only if its Lyapunov exponent satisfies $\lambda(x_0, \omega) < 0$. In general, the stochastic linear system (3), with ergodic perturbation, will have up to d Lyapunov exponents.

Model of the stochastic perturbation

To model the perturbation ξ_t, use is made of the results of Refs. [16–18], where it was shown that the Ornstein–Uhlenbeck process can be used to represent random phenomena present in electric power systems. Considering that in general those perturbations are restricted in size, the model used here consists of

$$\xi_t^\rho = \rho \cdot \sin(\eta_t), \quad \rho \geqslant 0, \tag{5}$$

where

- η_t is a stationary solution of the stochastic differential equation known as Ornstein–Uhlenbeck equation

  $$d\eta_t = -\alpha\eta_t dt + \beta dW_t \quad \text{in } \mathbb{R}^1. \tag{6}$$

  Here W_t denotes the standard 1-dimensional Wiener process. The parameters α and β must be estimated from real measurements of the phenomenon that one wants to model. In this paper we use $\alpha = \beta = 1$, a particular case of the perturbation model reported in Ref. [16].

- ρ is a parameter that models the amplitude of the effect of the perturbation, i.e. for $\rho = 0$ we have the unperturbed system (1).
- W_t denotes Brownian Motion.

Uniqueness of the Lyapunov exponent

Let us consider the stochastic linear system (3) with perturbation given by Eqs. (5) and (6), under the conditions reported in Ref. [21]. Then the system has a unique Lyapunov exponent for each perturbation size $\rho > 0$, given by

$$\lambda(\rho) = \limsup_{t\to\infty} \frac{1}{t} \log \left\|\varphi(t, x_0, \xi_t^\rho(\omega))\right\|, \tag{7}$$

for every initial condition $x_0 \in \mathbb{R}^d \setminus \{0\}$, with probability 1 (almost surely). This means, in particular, that the system (3) is asymptotically (and exponentially) stable with probability 1 for the perturbation of size ρ if and only if $\lambda(\rho) < 0$.