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Optimal power flow of a distribution system based on increasingly tight cutting planes added to a second order cone relaxation

S.Y. Abdelouadoud [a,b,*], R. Girard [a], F.P. Neirac [a], T. Guiot [b]

[a] MINES ParisTech, 1 Rue Claude Daunesse, F-06904 Sophia Antipolis Cedex, France
[b] Centre Scientifique et Technique du Batiment, 290, Route des Lucioles, F-06904 Sophia Antipolis Cedex, France

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ABSTRACT

Convex relaxations of the optimal power flow (OPF) problem have received a lot of attention in the recent past. In this work, we focus on a second-order cone (SOC) relaxation applied to an OPF based on a branch flow model of a radial and balanced distribution system. We start by examining various sets of conditions ensuring the exactitude of such a relaxation, which is the main focus of the existing literature. In particular, we observe that these sets always include a requirement on the objective to be a minimization of a function increasing with the branch flow apparent powers. We consider this hypothesis to be at odds with what is to be expected of an active distribution system and demonstrate in specific case studies its counterproductive impact. We continue by introducing an objective function allowing distributed generations and storages (DGS) to take advantage of the benefits they bring to the power system as a whole. As this entails the possibility for the relaxation not to be exact, we describe and prove the theoretical convergence to optimality of an algorithm consisting in adding an increasingly tight linear cut to the SOC relaxation. In order to allow the attainment of a solution satisfying the network constraints in a finite number of steps, we continue by introducing a tailored termination criterion. Afterwards, we investigate the ability of our algorithm to obtain a satisfactory solution on several case studies, spanning various network sizes, number of nodes equipped with DGs and their level of penetration. We then conclude on the benefits brought about by this approach and reflect on its limits and the opportunities for further improvements.

Introduction

With the aim to increase the sustainability of the electric power system, the share of renewable energies in the production mix is scheduled to increase in the future. For example, the European Union has set goals for its member states in order to attain a 20% share of renewable energy in its final energy consumption by 2020, and some countries have taken even more ambitious stances. This target will be partially met by integrating significant amounts of dispersed renewable energy generators (mainly photovoltaic (PV) and wind power) to the distribution grid. These developments will have a considerable impact on the design and operation of the electric system, both at the national and local level and so new tools will be needed to assist in the planning and operation of at least the distribution network. Indeed, as the current passive distribution network turns into an Active Distribution Network (ADN) (see [1] for a definition) with the introduction of partially and totally controllable generation and storage means, planning studies based solely on power flows for extreme load conditions will not be adapted anymore, as they would prevent the distribution system operators from taking advantage of the full benefits distributed generation could bring.

Considering the similarities between the current transmission network and the future ADN, it is a safe bet to assume that the optimal power flow, a framework first introduced in 1962 by [2] and now widely used for the planning and operation of the transmission network, will prove useful for this purpose. However, it has been remarked by several authors such as [3,4] that some characteristics inherent to the distribution system (in particular high R/X ratios and the radial nature of its topology) prevent us from applying traditional transmission system OPF algorithms, such as the Newton–Raphson method implemented in, for example, the Matpower package [5]. Moreover, simplifications commonly used in the planning of the transmission system, such as the linearization of the power flow constraints, are known to produce poor results with high R/X networks.

* Corresponding author at: MINES ParisTech, 1 Rue Claude Daunesse, F-06904 Sophia Antipolis Cedex, France.
E-mail address: seddik.abdelouadoud@mines-paristech.fr (S.Y. Abdelouadoud).

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In this context, the general objective pursued in this paper is to provide a methodology solving the single-stage OPF problem in a medium-voltage balanced radial distribution system to simulate DGS, with the purpose to integrate it in planning studies. This last conditions entails the need for a methodology that privileges speed of convergence and accuracy over precision, as it is expected to be run a large number of times to compare various planning options over long periods and with hour-long time steps, with significant uncertainties in input data. Additionally, its implementation should be versatile in terms of objective function and constraints to accommodate the uncertainties surrounding the future regulatory environment of the distribution system.

Of course, OPF in the distribution system have been envisioned for purpose other than the one described here, such as real time control [6] or optimal DG placement [7]. Nonetheless they all share an underlying common structure of Quadratically Constrained Quadratic Problem (QCQP) that we will describe below.

OPF as a QCQP

Traditionally, OPF based on bus injection model have been associated with meshed transmission systems and branch flow models have been used for radial distribution systems, see [8,9] for an up-to-date view on the subject. It has been recently proven that both can be cast as QCQP that are non-convex due to the presence of quadratic equality constraints, in the form of a rank constraint in the bus injection model [10] and power flow constraints in the branch flow model [11]. It is thus a NP-hard problem and cannot be solved in polynomial time while guaranteeing global optimality in general. A wide range of techniques have been employed including conjugate gradient, successive quadratic programming, branch-and-bound, Lagrange relaxation, interior point methods, simulated annealing, genetic algorithm and particle swarm optimization that represent various compromises between optimality and convergence speed and between versatility and tailoring for a specific problem (see [12] for a review of deterministic algorithms and [13] for non-deterministic ones). Considering our emphasis on convergence speed and versatility, and the parallel development of high performance SOC solvers and studies of quadratic convex relaxations of the OPF problem, we have chosen to focus our interest on them.

Convex relaxations

In the same manner that the equivalent QCQP of an OPF depends on which model is used, the corresponding convex relaxations will also depend on the model chosen.

1. Convex relaxation based on bus injection model are obtained by relaxing the rank constraint, as explained in [10]. We then obtain a semi-definite problem with a size proportional to the square of the size of the initial problem
2. Convex relaxation based on branch flow model. These relaxations are done in two steps. The first is a relaxation of bus angle constraints that is always exact for radial network [11] and the second relaxes quadratic equalities to convex inequalities to yield a SOC problem of a size equivalent to that of the original problem.

In the case of a radial network, [8] has shown that these relaxations are equivalent. Consequently, we choose the SOC option as the size of the resulting problem is smaller. This is supported by the findings in [8], where the authors have compared computational time to solve both relaxations on radial networks. While these were equivalent for small networks (9–39 nodes), the SOC relaxation was two orders of magnitude faster for the 300 bus test case.

Outline

For the remainder of this paper, we will focus on the SOC relaxation applied to a branch flow model of a balanced radial distribution system. We will start by stating a mathematical representation of the problem and present some of the conditions under which this relaxation may be exact, the focal point of most of the existing literature. We will then illustrate in which situations these conditions cannot be met on a simplified case study and introduce the cutting plane concept we will use in these instances, before proving its theoretical convergence to global optimality. We continue by introducing a termination criterion so that the algorithm is able to reach a satisfactory solution in a few iterations of a SOC solver, before analyzing its behavior on several case studies. We will conclude by reflecting on the applicability of the presented algorithm for our purpose and on the ways its performance and representativity of the problem can be enhanced.

The OPF model and its SOC relaxation

In the following, we will adopt the notations below:

Nomenclature

$j \in [1, J]$	the index of a node, where J is the node count and 1 is the root node
$f(j)$	the father node of node j, exists and is unique for all nodes except the root node
$c(j)$	the set of children nodes of node j
$P_{i,j}$	the active power flowing from node i to node j
$Q_{i,j}$	the reactive power flowing from node i to node j
$I_{i,j}$	the square of the current flowing from node i to node j
$r_{i,j}$	the resistance between node i and node j
$x_{i,j}$	the reactance between node i and node j
$z_{i,j}$	the impedance magnitude between node i and node j
$I_{i,j}^{max}$	the square of the maximal current that may flow from node i to node j
V_j	the voltage magnitude at node j
V_{OLTC}	the set of permissible voltages at the root node, dictated by the on-load tap changer characteristics
P_j^{st}	the active power injected by the storage inverter at node j
$P_j^{st,sp}$	the set point for the active power injected by the storage inverter at node j
Q_j^{st}	the reactive power injected by the storage inverter at node j
$S_j^{max,st}$	the maximal apparent power of the storage inverter at node j
P_j^{pv}	the active power injected by the PV inverter at node j
$P_j^{pv,sp}$	the set point for the active power injected by the PV inverter at node j
Q_j^{pv}	the reactive power injected by the PV inverter at node j
$S_j^{max,pv}$	the maximal apparent power of the PV inverter at node j
P_j^{load}	the active power load at node j
Q_j^{load}	the reactive power load j
$\mathbf{x}$	the vector of control and state variables, that thus includes active and reactive flows in the lines, voltage magnitudes at each nodes and active and reactive injections of storage and PV inverters
$\Delta(\mathbf{x})$	the convex quadratic objective function representing the interests of the DGS operators