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Using Shapley value in multi-objective data envelopment analysis: Power plants evaluation with multiple frontiers

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ABSTRACT

Data envelopment analysis (DEA) is a well-known non-parametric methodology for measuring the relative efficiency of decision making units (DMUs). One of the points that must be considered in the usage of DEA is relationship between the number of DMUs and the number of inputs/outputs. Managers wish to apply a large set of inputs/outputs as well as classify them in order to calculate a relative technical efficiency. But, the limitations of the conventional models do not allow that have been used a large set of inputs and outputs. On the other hand, removing a specific input (or output) can greatly change the evaluation results. This paper introduces a multi-objective DEA (MODEA) model to remove the limitations of the conventional DEA models. In MODEA model, multiple objectives generate multiple frontiers. Despite other multi-objective problems, the existing methods and procedures for obtaining Pareto solutions may not be useful in MODEA because, we do not need Pareto solutions. Therefore, appropriate and fair weights regarding to the importance of each objective should be found to obtain unique efficiency score. For this purpose, the Shapley value as cooperative game has been applied. The results of the proposed approach are closer to reality. The approach regardless of the number of DMUs discriminates among the DMUs more effectively. In addition, DMUs are compared by multi-category of measures in the competitive environment. Furthermore, the abilities of the approach are demonstrated by a case study of the Iranian power plants. Also, the case study shows the steps of solving MODEA model.

Introduction

Data envelopment analysis (DEA) is a non-parametric methodology to measure the relative efficiency of DMUs. Farrell [7] first used DEA to measure technical efficiency for a set of organizations, but this idea has been developed and extended by Charnes et al. [3]. Since the introducing of DEA, there has been an impressive growth in theory and applications. The researchers have applied it in different areas such as health care units, university departments, bank branches, military operations, criminal courts, information system projects, power plants, transportation systems, mining operations and manufacturing processes.

Suppose we have n DMUs, where each DMU_j ($j = 1 \ldots n$) produces s outputs y_{rj} ($r = 1 \ldots s$) by utilizing m inputs x_{ij} ($i = 1 \ldots m$). According to these notations, DEA uses the following model for evaluating of DMU_o's efficiency:

$$\begin{aligned} Max \quad & \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ s.t. \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leqslant 1, \quad j = 1, \ldots, n \\ & u_r, v_i \geqslant \varepsilon, \quad i = 1, \ldots, m, \quad r = 1, \ldots, s \end{aligned} \tag{1}$$

where u_r and v_i are the weight of inputs and outputs and $\varepsilon > 0$ is a non-Archimedean number that is smaller than any positive real number. Model (1) is run for each DMU that is represented by "o". The relative efficiency of DMUs is calculated by assigning 1 for efficient DMUs and less than 1 for inefficient ones. DEA uses the combinations of multiple inputs that produce multiple outputs. A guideline commonly applied [9] is that the number of DMUs should be greater than triple of the total number of input and output variables ($3(m + s) < n$).

When the large scale of variables (inputs/outputs) is available, aggregating variables is commonly used to overcome to this problem to reduce the number of inputs/outputs. For this purpose, various approaches are applied in literatures such as principal component analysis, factor analysis and neural networks. These approaches make some difficulties in target setting and evaluation goals because, original inputs and outputs are usually converted to

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new variables so that the latter variables are not as type as the former. It means that these new variables cannot be appropriate for analysis and target setting.

The first time, Banker [1] used game theory to DEA in which efficiency measurement using DEA was shown to be equivalent to a two-person zero-sum finite game. In the paper, one player was the DMU to be assessed and the other player was an external evaluator. Banker et al. [2] reformulated the pervious approach as a constrained game to take into account the non-zero slacks that may exist. Afterward, Chen et al. [4] have considered the efficiency game between two supply chains members. They have shown that there are numerous Nash equilibriums efficiency plans for the supplier and the manufacturer with respect to their efficiency functions. Also, a bargaining model has been proposed to analyze the decision process and to determine the best efficiency plan strategy in the central as well as the decentralized control cases. Another game model called Egoist's dilemma has been mentioned in Nakabayashi and Tone [15]. The authors considered multi criteria problems of consensus-making among organizations for evaluating performance when the players are supposed to be egoistic. They used cooperative game theory and proposed a solution for this problem.

Tchangani [18] analyzed the performance of a production unit in two directions: resource utilization versus output performance. They proposed the satisfiability functions in the framework of satisficing game theory for evaluating performance of each decision making unit. Liang et al. [13] have proposed a specific paper that generalized the original DEA cross-efficiency to game cross-efficiency. They considered each DMU as a player that seeks to maximize its own efficiency. Also, an algorithm for obtaining the best scores has been presented in the study. Wu and Liang [19] proposed an approach to evaluate and rank alternatives in multi criteria decision making via an extension of DEA. They improved and completed Liang et al. [13] algorithm in which each alternative is considered as a player who seeks to maximize its own score. Li and Liang [12] used Shapley value index for deriving the importance of variables in DEA models. They have defined an efficiency change ratio to calculate the impact of each input/output on the efficiency score. Then, a characteristic function of each coalition based upon efficiency change ratio has been defined. Du et al. [5] have been used a Nash bargaining game model to measure the efficiency of DMUs that have two-stage network structures or processes. Under Nash bargaining theory, the DEA efficiency model is a cooperative game model and the two stages are viewed as players. Lozano [14] has presented the newer work in this field. The author proposed a cooperative DEA-game based on the idea that different organizations can gain if they share data on the input consumption and output production. Two types of DMUs have been considered: observed and planned. The inputs and outputs for observed DMUs are known but the planned DMUs correspond to operation points whose desired outputs levels are known and for which the minimum inputs costs need to be computed. Jahangoshai Rezaee et al. [10] used the bargaining game for measuring the operational and non-operational performance of thermal power plants in Iran. The case study used in this paper is to explore how two categories of measures are integrated for measuring performance. Jahangoshai Rezaee et al. [11] provided an integrated model for evaluating the performance of units with using DEA and bargaining game. They classified inputs and outputs into two categories including medical and geographical aspects and applied the proposed approach for measuring the performance of Tehran health centers.

In this paper, a multi-objective DEA is introduced to overcome these difficulties by using a large set of inputs and outputs. Inputs and outputs classify in different categories. Afterward, Shapely value has been used as a criterion to determine the efficacy of categories (objectives). Then, unified efficiency scores are calculated by using Shapely values. There are few researches in the field of combined DEA and game theory. The main motivations of this paper are: (1) Introducing multi-objective DEA with multi-category inputs and outputs as well as discussing on the reasons of using MODEA instead of single-objective DEA. (2) Applying Shapley value as the solution of the cooperative game to determine objective functions efficacies. (3) Solving MODEA for obtaining unified efficiency by considering objectives efficacies.

The continuation of this paper is organized as follows: section 'Pitfalls in DEA and Multi-objective data envelopment analysis' provides some pitfalls in DEA as well as introduces the multi-objective DEA as a new approach for evaluating DMUs. Introducing a measure based on Shapley value to determine objectives efficacies is given in section 'Shapley value in MODEA'. Section 'Case study and analyses' presents the case study of power plants to show the abilities of the proposed approach. The results and analyses of case study are provided in this section. Section 'MODEA affords opportunities for more analysis' provides more analysis regarding to MODEA model and gives interesting outcomes. Finally, summary and conclusion are given in the last section.

Pitfalls in DEA and multi-objective data envelopment analysis

The DEA studies are classified as: models, data set of the DMUs' values and returns to scale assumption. In a real application, various measures may be used to evaluate DMUs. These measures are categorized as technical, business, economic, R&D, social, managerial, environmental, safety. However, if all measures are considered, the existing models may be failed because they have limitations for evaluating DMU with the large set of inputs/outputs. DEA as a powerful methodology for evaluating and performance assessment, is sensitive to the number of DMUs as well as the number of inputs/outputs variables.

If DMUs are taken from infinite set, then DEA models can provide the best estimation of the production frontier. In other words, with decreasing in the number of DMUs, the error of the production frontier estimation increases, and the possibility of domination for each DMU decreases by other efficient DMUs. Therefore, the numbers of efficient DMUs increase, while some of them are not really efficient. In practice, a large number of units are not available. There are sufficient units in some cases such as bank branches or schools whereas; there is no access to a large number of units in many other cases such as power plants. In other words, there are many inputs and outputs that decision makers are interested in using them in evaluation, but the number of DMUs is not sufficient. Therefore, establishing equilibrium between the number of DMUs and the number of inputs and outputs is an important factor. Friedman and Sinuany-Stern [9] have proposed a guideline for DEA. Based on this protocol, the number of DMUs should be greater than the triple of the total number of input and output variables ($3(m+s) < n$). More discussions on the effect of the number of DMUs and the number of inputs/outputs in the production frontier can be found in Fried et al. [8]. Furthermore, in reality, since most of the used inputs/outputs are dependent to each other for each DMU, then they are often correlated. Dyson et al. [6] showed that the omission of a highly correlated variable can have a significant impact on the efficiency scores of some production units. They have been shown that even removal of a highly correlated input (or output) can greatly change the evaluation results. Moreover, the removal of correlated data may not be rational in the evaluations. Often the decision makers wish to include many indicators in order to present a relative evaluation. To overcome these problems, a new approach is introduced in this paper. Based on this approach, a multi-objective DEA (MODEA) is combined with game theory.