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A method for evaluating the accuracy of power system state estimation results based on correntropy

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ABSTRACT

Power system state estimation (SE) is a crucial basic function in energy management systems (EMSs) which offers the basic load flow models. It is a critical problem to check whether the SE results are credible enough to be used for online decision making or close-loop control. However, there are few published works focus on this topic till now. The accuracy of SE results can be affected by various factors, many of them are very difficult to quantify. Hence a feasible method is to estimate the accuracy of SE results based on residuals. Due to the huge number of measurements in real power systems, it is necessary to establish a reasonable scalar index that represents the residuals of all measurements and directly indicates the accuracy of the SE results. This paper provides a systematic research on the SE accuracy evaluation problem and proposes a new SE accuracy evaluation index based on correntropy. Some conventional SE accuracy evaluation indices are also introduced and compared theoretically and also through extensive numerical tests.

1. introduction

There are mainly three essential problems evolved in power system state estimation: measurement accuracy quantification, SE algorithms and result accuracy evaluation. Extensive researches focus on SE algorithms have been published, especially on the suppression of gross errors in topology [1,2], analog measurements [3–6] and network parameters [7–9]. Some effective studies on the measurement accuracy quantification have also been reported [10,11]. However, there are few systematic researches focus on the SE results accuracy evaluation problem. Since the performances of most functions in EMS depend on the SE results, it is a critical problem to check whether the SE results are credible enough to be used for online decision making or close-loop control.

The accuracy of the SE results can be affected by various factors including errors in PT/CT, AC sampling errors, errors from information channels, device malfunctioning and parameter errors, SE algorithm, etc. Many of them are very difficult to quantify. Hence it is very difficult to give a deductive combination for all these factors to evaluate the accuracy of SE results, and a residual-based accuracy estimation method should be employed instead. Generally speaking, the smaller the measurement residuals are, the more accurate the SE result is. Since there are huge number of measurements in a real power system, the various measurement residuals must be represented by a scalar index to quantify the SE accuracy intuitively.

There are several conventional SE accuracy evaluation indices. The most common method is using the first or second order norms of residual vector to build evaluation indices. Such indices sum up the absolute values or square values of the residual vector [12,13]. These indices lack clear physical meanings, and maybe infected by gross errors. In metrology, measurement uncertainty is a textbook method to evaluate the accuracy of SE [14,15]. The calculation of measurement uncertainty is a deductive method, which is very difficult for implementation in large-scale practical power systems. Refs. [16,17] suggest using a threshold value of 3% or 5% of the measurement value as a confidence bound for each measurement. In China, the State Grid Corporation of China (SGCC) proposes an official standard for evaluating the accuracy of the SE, which named as acceptance rate (*AR*). *AR* counts the rate for the measurement whose residual is less than a artificial set threshold value. In theoretically, *AR* is loosely connected with measurement uncertainty, but its crucial threshold values are totally determined by manual experience.

This paper provides a systemic research on the problem of SE result accuracy evaluation, but the measurement accuracy quantification and SE algorithms are beyond this paper. An important contribution of this paper is that it proposes a new SE accuracy evaluation method, the scalar index of correntropy (*COE*), and

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makes a comprehensive comparison of the proposed *COE* with several common scalar indices, including the mean absolute value of weighted residuals (*MAR*), mean squares of the weighted residuals (*MSR*), and acceptance rate (*AR*).

The remainder of this paper is organized as follows. Section 2 provides an introduction to the traditional indices. The correntropy index is introduced in Section 3. Section 4 introduces the comparison method used in this paper. Extensive numerical tests used to verify the performances of the various indices, and their results are described in Section 5.

2. Common scalar indices

2.1. Mean absolute value of weighted residuals (MAR)

The mean absolute value of weighted residuals (*MAR*) can be expressed as:

$$MAR = \frac{1}{m}\sum_{i=i}^{m}|r_i/\delta_i| \tag{1}$$

where m denotes the number of measurements, r_i and δ_i are the measurement residual and standard deviation for the ith measurement. A smaller *MAR* index indicates more accurate SE results.

2.2. Mean squares of weighted residuals (MSR)

The mean squares of weighted residuals (*MSR*) can be defined as:

$$MSR = \frac{1}{m}\sum_{i=i}^{m}(r_i/\delta_i)^2 \tag{2}$$

A smaller *MSR* index indicates more accurate SE results.

2.3. Measurement uncertainty and acceptance rate (AR)

Measurement uncertainty is a textbook method to measure the accuracy of the SE results in metrology. The International Standards Organization (ISO) has defined two types of measurement uncertainties: standard and extended. Standard measurement uncertainty U_{ISO} is defined as [15]:

$$U_{ISO} = \pm K[U_A^2 + U_B^2]^{1/2} \tag{3}$$

where U_A is type A uncertainty, which is measured by statistical methods, and U_B is type B uncertainty, which measured more subjectively using non-statistical methods. K is a multiplier used to obtain the confidence of interest. The calculation of measurement uncertainty is deductive which depends on the measurement standard deviations and artificial justice. In real power systems, however, the measurement standard deviations comprise various factors and it is very difficult to estimate their standard deviations. Although some control centers have their manually specified values for measurement standard deviation, δ_i, they are usually inaccurate. The quantification of type B uncertainty for large-scale real power systems is even more difficult. Hence, the application of standard measurement uncertainty becomes impractical.

Instead of standard measurement uncertainty, extended measurement uncertainty can be used to evaluate the accuracy of the SE results. Extended measurement uncertainty is defined as:

$$P(|r_i| < kU_{ISO}) = \gamma \tag{4}$$

where k is the coverage factor and γ is the confidence level, which can be 99%. Assume that the measurement errors are Gaussian distributed, the probability of the measurement errors falling in the interval $[-U_{ISO}, +U_{ISO}]$ is 68.3%, and the probability of the measurement errors falling in the interval $[-3U_{ISO}, +3U_{ISO}]$ is 99.7%. The corresponding coverage factor k is about 3.0 for $\gamma = 99\%$. For a reasonable SE result, most measurement residuals must fall into the interval defined by the extended measurement uncertainty with an appropriate confidence level or coverage factor. Therefore, the acceptance rate (*AR*) index can be used to evaluate the accuracy of the SE results:

$$AR = \frac{1}{m}\sum_{i=1}^{m}\alpha_i \times 100\% \tag{5}$$

where

$$\alpha_i = \begin{cases} 1 & \text{if } |r_i| < \varepsilon_i \\ 0 & \text{else} \end{cases} \tag{6}$$

$\varepsilon_i = kU_{ISOi}$ stands for the extended measurement uncertainty and defines the confidence interval. The *AR* index in (6) has been adopted by the SGCC as an official standard for evaluating the SE accuracy. However, since the standard measurement uncertainty U_{ISO} is difficult to estimate in practical power systems, the value of the extended measurement uncertainty ε_i is also difficult to quantify, and is always specified according to manual experience. Although the acceptance rate is derived from measurement uncertainty theory, it is very subjective since the threshold value ε_i is set manually.

3. Correntropy based scalar index

3.1. Brief introduction

Correntropy is a generalized similarity measure between two random variables in signal processing [18–20]. Assume that x_1 and x_2 are two random variables with probability density functions f_1 and f_2, respectively. Their Renyi's quadric correntropy can be defined as:

$$\begin{aligned} &H(x_1, x_2) = -\log I \\ &I = \int f_1(x)f_2(x)\mathrm{d}x \end{aligned} \tag{7}$$

where I is the cross information potential and $H(x_1, x_2)$ is Renyi's quadric correntropy of x_1 and x_2. The probability density function can be estimated from the kernel function based on the samples, and the cross information potential can be estimated as:

$$\hat{I} = \frac{1}{m}\sum_{i=1}^{m}\kappa(x_{1i}, x_{2i}) \tag{8}$$

where m is the number of samples, x_{1i} and x_{2i} are the ith sample values for x_1 and x_2, respectively, and κ is the kernel function, which should be symmetric non-negative definite. According to the Moore–Aronszajn theorem [21,22], for any symmetric non-negative definite function κ, there exists a corresponding reproducing kernel Hilbert space F defined by mapping φ: $X \to F$ with κ as a kernel function. This yields:

$$\langle\varphi(x_1), \varphi(x_2)\rangle = \kappa(x_1, x_2) \tag{9}$$

where $\langle\cdot\rangle$ stands for the inner product, which measures the information potential in Hilbert space. Hence, one can conclude that correntropy in fact maps the random variables from their original space to another Hilbert space in which their cross information potential is calculated. An interesting characteristic is that all of the reproducing kernel Hilbert spaces with definite dimensions are omorphism, so we can choose any symmetric non-negative definite kernel function to construct the reproducing kernel Hilbert space F. The entire analysis can be carried out in space F without knowing its exact meaning.

The most commonly used kernel function is the Gaussian kernel: