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Enhanced Lagrangian relaxation solution to the generation scheduling problem

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ABSTRACT

This paper proposes an enhanced Lagrangian relaxation (LR) solution to the generation scheduling problem of thermal units, known as unit commitment (UCP). The proposed solution method is characterized by a new Matlab function created to determine the optimal path of the dual problem, in addition, the initialization of Lagrangian multipliers in our method is based on both unit and time interval classification. The proposed algorithm is distinguished by a flexible adjustment of Lagrangian multipliers, and dynamic search for uncertain stage scheduling, using a Lagrangian relaxation–dynamic programming method (LR–DP). After the LR best feasible solution is reached, a unit decommitment is used to enhance the solution when identical or similar units exist in the same system. The proposed algorithm is tested and compared to conventional Lagrangian relaxation (LR), genetic algorithm (GA), evolutionary programming (EP), Lagrangian relaxation and genetic algorithm (LRGA), and genetic algorithm based on unit characteristic classification (GAUC) on systems with the number of generating units in the range of 10–100. The total system production cost of the proposed algorithm is less than the others especially for the larger number of generating units. Computational time was found to increase almost increases linearly with system size, which is favorable for large-scale implementation.

1. Introduction

Unit commitment problem (UCP) is a nonlinear, mixed integer combinatorial optimization problem. It is defined as the problem of how to schedule generators economically in a power system in order to meet the requirements of load and spinning reserve. Usually this problem is considered over some period of time, such as the 24 h of a day or the 168 h of a week. It is a difficult problem to solve in which the solution procedures involve the economic dispatch problem as a sub-problem.

Since the problem was introduced, several solution methods have been developed. However, they differ in the solution quality, computational efficiency and the size of the problem they can solve. These methods or approaches have ranged from highly complex and theoretically complicated methods to simplified methods.

In the past, various approaches such as DP [1], branch-and-bound B&B [2] and Lagrangian relaxation (LR) [3] were proposed for solving the UCP. However, not all of these methods are regarded as feasible and/or practical as the size of the system increases.

For moderately sized production systems, exact methods, such as dynamic programming (DP) or (B&B) [2] can be used to solve the UCP, successfully. For larger systems, exact methods fail because the size of the solution space increases exponentially with the number of time periods and units in the system. As a result, the computation time of exact methods becomes impractical. In these cases heuristic methods (evolutionary programming (EP), Tabu Search (TS), Simulated Annealing (SA), Genetic Algorithms (GA), etc.) can be used to produce near optimal solutions in a reasonable computation time. For heuristic methods optimality is not given such a high priority but the emphasis is on finding good solutions in a short time. This often results in the solution method being more simple and transparent than exact solution methods [4].

The application of LR in the scheduling of power generations was proposed in the late 1970s. These earlier methods used LR to substitute the common linear programming (LP) relaxation approach as a lower bound in the B&B technique [5]. In this regard, great improvement of computational efficiency was achieved compared with previous B&B algorithms.

In recent years, methods based on LR, have become the most dominant ones. This approach has shown some potential in dealing with systems that consist of hundreds of generating units and is motivated by the separable nature of the problem, and several examples have been reported in the literature.

Based on the sharp bound provided by the Lagrangian dual optimum, it is expected that a sub-optimal feasible solution near the dual optimal point can be accepted as a proper solution for the primal problem. A more direct and fairly efficient methodology which has used this idea was presented in [6] by Merlin, for UCP using LR method and validated at Electricite De France. Due to its reasonable performance, the successive improvement of the LR algorithm,

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in the last few years, has mainly followed the work in [6]. The problem which is supposed to be handled by this algorithm consists of thermal units only.

In [7], they combined LR, sequential UC based on the least reserve cost index and unit decommitment (UD) based on the highest average spinning reserve cost index. However, this method could not decommit some units that violate the minimum up time constraints even though the excessive reserve exists, leading to a higher production cost.

In the advent of heuristic approaches, GA [8], EP [9], SA [10], and TS [11] have been proposed to solve the UC problems. Nevertheless, the obtained results by GA, EP, and SA required a considerable amount of computational time especially for a large system size. There was an attempt to combine the LR and GA method (LRGA) to obtain a higher quality of UC solution in a shorter time by using normalized Lagrange multipliers as the encoded parameter [12].

2. Unit commitment problem formulation

The objective of the UCP is to minimize the system operating costs, which is the sum of production and startup costs of all units over the entire study time span (e.g., 24 h), under the generator operational and spinning reserve constraints. Mathematically, the objective function, or the total operating cost of the system can be written as follows:

$$J = \min_{P_i^t u_i^t} f(P_i^t, u_i^t) = \min_{P_i^t u_i^t} \left(\sum_{t=1}^{T} \sum_{i=1}^{N} u_i^t \left[F_i(P_i^t) + S_i^t (1 - u_i^{t-1}) \right] \right) \quad (1)$$

Subject to:

(1) The startup cost is modeled by the following function of the form:

$$S_i^t = \begin{cases} HS^i, & \text{if} \quad X_{off,i}^t \leqslant T_i^{down} + CH^i \\ CS^i, & \text{if} \quad X_{off,i}^t > T_i^{down} + CH^i \end{cases} \quad (2)$$

(2) Power balance

$$\sum_{i}^{N} u_i^t P_i^t = D^t \quad (3)$$

(3) Spinning reserve requirements:

$$\sum_{i}^{N} u_i^t P_i^{\max} \geqslant D^t + R^t \quad (4)$$

Generating limits:

$$u_i^t P_i^{\min} \leqslant P_i^t \leqslant u_i^t P_i^{\max} \quad (5)$$

Minimum up time constraint:

$$\left(X_{on,i}^{t-1} - T_i^{up} \right)(u_i^{t-1} - u_i^t) \geqslant 0 \quad (6)$$

$$X_{on,i}^t = (X_{on,i}^{t-1} + 1) u_i^t \quad (7)$$

Minimum down time constraint:

$$(X_{off,i}^{t-1} - T_i^{down})(u_i^t - u_i^{t-1}) \geqslant 0 \quad (8)$$

$$X_{off,i}^t = (X_{off,i}^{t-1} + 1)(1 - u_i^t) \quad (9)$$

Fuel cost function $F_i(P_i^t)$ is frequently represented by the polynomial function:

$$F_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2 \quad (10)$$

where P_i^t is the output power of unit i at period t (MW), $F_i(P_i^t)$ is fuel cost of unit i when its output power is P_i^t ($), S_i^t is startup price of unit i at period t ($), u_i^t is commitment state of unit i at period t ($u_i^t = 1$: unit is on-line and $u_i^t = 0$ unit is off-line), N is total number of generating units, T is total number of scheduling periods, a_i, b_i, c_i are coefficients for the quadratic cost curve of generating unit i, $X_{off,i}^t, X_{oni}^t$ are number of hours the unit has been off-line/on-line (h), X_i^0 is initial condition of a unit i at $t = 0$, $X_i^0 > 0$: on-line unit, $X_i^0 < 0$: off-line unit (h), T_i^{up} is minimum up time (h), T_i^{down} minimum down time (h), HS^i, CS^i are the unit's hot/cold startup cost ($), CH^i is the cold start hour (h), D^t is customers' demand in time interval t, R^t is the spinning reserve requirements;

3. An improved flexible Lagrangian relaxation technique

In the Lagrangian relaxation approach, the system operating cost function of Eq. (1) of the unit commitment problem is related to the power balance and the spinning reserve constraints via two sets of Lagrangian multipliers to form a Lagrangian dual function.

$$L(P, u, \lambda, \mu) = f(P, u) + \sum_{t=1}^{T} \lambda^t \left(D^t - \sum_{i=1}^{N} u_i^t P_i^t \right) + \sum_{t=1}^{T} \mu^t \left(D^t + R^t - \sum_{i=1}^{N} u_i^t P_i^{\max} \right) \quad (11)$$

The LR procedure solves the UCP through the dual problem optimization procedure attempting to reach the constrained optimum.

The dual procedure attempts to maximize the Lagrangian with respect to the Lagrangian multipliers λ^t and μ^t, while minimizing it with respect to the other variables P_i^t, u_i^t subject to the unit constraints in Eq. (5) through Eq. (9). The dual problem is thus the search of the dual solution (Q) expressed as:

$$Q = \max_{\lambda^t, \mu^t} \left(\min_{P_i^t, u_i^t} L(P, u, \lambda, \mu) \right) \quad \lambda^t \geqslant 0 \text{ and } \mu^t \geqslant 0 \quad (12)$$

The Lagrangian function of Eq. (11) is rewritten as

$$L(P, u, \lambda, \mu) = f(P, u) - \sum_{t=1}^{T} \lambda^t \sum_{i=1}^{N} u_i^t P_i^t - \sum_{t=1}^{T} \mu^t \sum_{i=1}^{N} u_i^t P_i^{\max} + \sum_{t=1}^{T} \lambda^t D^t + \sum_{t=1}^{T} \mu^t (D^t + R^t) \quad (13)$$

When the Lagrangian multipliers $\lambda^{t(k)}$ and $\mu^{t(k)}$ are fixed for iteration k, the last two terms of the Lagrangian in Eq. (13) are constant and can be dropped from the minimization problem. Hence, the system (coupling) constraints can be relaxed and the search for the dual solution can be done through the minimization of the Lagrangian as:

$$\min_{P_i^t, u_i^t} L(P, u, \lambda^{(k)}, \mu^{(k)}) = \min_{P_i^t, u_i^t} \left(\sum_{t=1}^{T} \sum_{i=1}^{N} u_i^t \left\{ F_i(P_i^t) + S_i^t (1 - u_i^{t-1}) - \lambda^{t(k)} P_i^t - \mu^{t(k)} P_i^{\max} \right\} \right) \quad (14)$$

Then, the minimum of the Lagrangian function is solved for each generating unit over the time horizon, that is