ELSEVIER

Electrical Power and Energy Systems 28 (2006) 223–231

ELECTRICAL POWER & ENERGY SYSTEMS

www.elsevier.com/locate/ijepes

Optimal PMU placement for full network observability using Tabu search algorithm

Jiangnan Peng [a], Yuanzhang Sun [b], H.F. Wang [c,*]

[a] China Power Investment Corporation, Beijing, People's Republic of China
[b] Department of Electrical Engineering, Tsinghua University, Beijing, People's Republic of China
[c] Department of Electronic and Electrical Engineering, University of Bath, 39 Burnt House Road, Bath BA2 2AQ, UK

Received 4 April 2003; received in revised form 17 May 2005; accepted 25 May 2005

Abstract

This paper presents a fast analysis method for power system topology observability. The method is based on the linearized power system state estimator model and uses augmented incidence matrix. In the paper, the Optimal PMU (phasor measurement units) Placement (OPP) problem is formulated as to minimize the number of PMU installation subjecting to full network observability and enough redundancy. A global optimization algorithm, Tabu search, is proposed to solve the combinatorial optimization problem and a priority list based on heuristic rule is embedded to accelerate optimization. The effectiveness and flexibility of the proposed algorithms are demonstrated by numerical results tested in IEEE 14, IEEE 57 and NE 39 bus systems.

Keywords: Network observability analysis; Optimal PMU placement (OPP); State estimation; Tabu search (TS)

1. Introduction

In recent years, applications of phasor measurement units (PMUs) have been attracting more and more attentions in power systems security monitoring and control. This is due to the advantages that PMUs can offer real-time synchronized phasor measurements (voltage, current, powers, frequency etc.) contrary to the conventional SCADA measurement devices [1,2]. The prerequisite for an efficient and accurate control is the development of adequate meter placement scheme, which can realize the network full observability.

As we know, observability analysis is a fundamental component of real-time state estimation, which acts as the back bone of EMS applications. The theory of network observability can be divided into two main classes of algorithms: numerical and topological methods. The topological methods are based on whether a spanning tree of full rank can be constructed. The numerical methods rely on whether the measurement information gain or Jacobian matrix is of full rank. In this area, a lot of interesting work has been reported [3–9] and each of them has its own advantages and limitations. Conventionally, numerical methods involve huge matrix manipulation, and are computationally expensive. Moreover the accuracy of solution is apt to suffering from the computation error. In theory, if all nodes in power system have been installed with PMUs, then the whole system state is fully observable. However, considering the cost of the equipment together with communication links, optimal PMU placement (OPP) problem demands to reduce the number of PMU installation and concerns about where and how many PMUs should be implemented to a power system to achieve full-state observability at minimal cost. This problem began being addressed recently and certain progress has been achieved [10–13].

This paper introduces a novel topological method based on the augment incidence matrix proposed in [13] and the Tabu Search (TS) [14–18]. By doing so, solution of the combinatorial OPP problem requires less computation and is of higher robustness. The method is much faster and more convenient than the conventional observability analysis method using complicated matrix analysis. The paper is organized as follows. Section 2 presents the proposed observability analysis method based on incidence matrix for PMU applications and derives a linear phasor estimation model benefited from the definition of defining different measurement layers. In Section 3, the OPP problem is

* Corresponding author.
E-mail address: wanghf60@hotmail.com (H.F. Wang).

0142-0615/$ - see front matter © 2005 Published by Elsevier Ltd.
doi:10.1016/j.ijepes.2005.05.005

formulated and TS algorithm is applied to solve the combined optimization problem. Three IEEE test system results are given in Section 4 and the work is concluded in Section 5.

2. Observability analysis using augment incidence matrix for PMU application

Observability is defined as the ability to uniquely estimate the states of a power system using given measurements. It is well-known that the state estimation can not work even if measurements are redundant. Observability analysis is required to decide meter placement in order to maintain solvability of the observation equations in various conditions. Implementation of PMU presents an opportunity for improving observability analysis and state estimation. Linear phasor estimation model

Considering a power system with n nodes, the output of the state estimation is $2n-1$ dimensional state vector $X=[V_1,\ldots,V_n,\ \theta_1,\ldots,\theta_{n-1}]^{\mathrm{T}}$ containing the voltage magnitudes and phasor angle. The measurement equation can be expressed as [2]

$$\mathbf{Z} = \mathbf{h}(\mathbf{x}) + \boldsymbol{\varepsilon} \tag{1}$$

where measurement vector $\mathbf{Z}=[Z_1,Z_2,\ldots,Z_m]^T$, $\mathbf{Z}\in R^m$; measurement function $\mathbf{h}(\mathbf{x})$, $\mathbf{h}$:$R^{2n-1}\rightarrow R^m$; measurement error vector $\boldsymbol{\varepsilon}\in R^m$.

Since PMU can directly measure phasor, in order to establish the fast phasor estimator model, we select the optimal candidate measurement of essential measurements as the PMU correlative measurement sets (CMS). CMS includes measurements of the voltage phasor at a bus where a PMU is placed and all the current phasors incident to the bus. CMS, denoted by $\mathbf{Z}_{\mathrm{cor}}$, can be expressed as

$$\mathbf{Z}_{\mathrm{cor}} = \{\mathbf{Z}_i\}, \quad \mathbf{Z}_i = [\dot{V}_i, \dot{I}_{i_1}, \ldots, \dot{I}_{i_l}]^{\mathrm{T}} \quad i = 1, \ldots, n_{\mathrm{p}} \tag{2}$$

where n_{p} is the number of PMU placement and, i_l is the total number of branches incident to bus i. Hence, n_{p} PMU can directly provide $2n_{\mathrm{p}}$ bus voltage and the corresponding i_l current phasor at all the measurement node i. Obviously, the CMS can be represented as $\mathbf{Z}_{\mathrm{cor}}=[\mathbf{Z}_{\mathbf{v}}\mathbf{Z}_{\mathbf{I}}]^{\mathrm{T}}$, where $\mathbf{Z}_{\mathbf{v}}$, $\mathbf{Z}_{\mathbf{I}}$ are bus voltage phasor and branch current phasor, whose dimensions are $m_{\mathrm{V}}=n_{\mathrm{p}}$ and

$$m_1 = \sum_{i=1}^{n_p} i_l$$

respectively. Since, the PMU measurement based on GPS synchronism technology is far more accurate than traditional analog measurements based on SCADA, we can neglect the error and choose directly the PMU measurement values as the estimate value. This yields the simple linear observation model. For the PMU direct measured voltage $\mathbf{V_M}$ and the adjacent non-measured voltage $\mathbf{V_C}$, Eq. (1) can be rewriten as follows [2]

$$\begin{bmatrix} \mathbf{Z}_{\mathrm{V}} \\ \mathbf{Z}_{\mathrm{I}} \end{bmatrix} = \mathbf{Hx} = \begin{bmatrix} \mathbf{I} & 0 \\ \mathbf{M}_{\mathrm{IB}} Y_{\mathrm{BB}} A_{\mathrm{MB}}^{\mathrm{T}} & M_{\mathrm{IB}} Y_{\mathrm{BB}} A_{\mathrm{CB}}^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{V_M} \\ \mathbf{V_C} \end{bmatrix} \tag{3}$$

where, $\mathbf{M_{IB}}$ is $m_l \times b$ measure-branch incidence matrix associated with the current phasor measurements, $\mathbf{Y}_{\mathrm{BB}}$ is $b\times n$ diagonal matrix of branch admittances, $\mathbf{A}_{\mathrm{MB}}$ and $\mathbf{A}_{\mathrm{CB}}$ are measured and non-measured node-branch incidence submatrices, respectively.

Conventional observability analysis can check the satisfaction of the following condition

$$\mathrm{rank}(\mathbf{H}) = 2n-1 \tag{4}$$

The above condition means that the measurement matrix is full rank. However, it is difficult to directly assess $\mathbf{H}$ due to its massive dimension. Generally, an arbitrary PMU placement set cannot guarantee the satisfaction of Eq. (4). Moreover, Eq. (4) can not provide the quantitative index for analyzing different PMU placement scheme. Hence, it is necessary to develop a quantitative assessment technique to guide the selection of PMU placement sets based on the following observability conditions:

Condition 1. If one end bus voltage and branch current phasor of a branch is known, then the other end bus voltage phasor can be calculated via branch equation.

Condition 2. For a branch with known voltage phasor at its both ends, its current phasor can be calculated.

Condition 3. For a zero-injection node without a PMU placement, if just only one of the incidence branch current is unknown, then the current can be calculated by KCL law.

Condition 4. When all the bus voltages of an unknown zero-injection node are known, the voltage can be computed by corresponding node voltage equation.

Based on these conditions above, it is convenient to classify the measurements, from the topological point of view, into three layers, the measurement, pseudo-measurement, and the extension-measurement. The states variables connected directly to PMUs buses (i.e. the CMS) can be defined as the measurement layer variables. Similarly, pseudo-measurement variables are voltages or currents those can be calculated by CMS via Ohm law (conditions 1 and 2 above) and extension-measurement variables are the voltages or currents that can be inferred by using the KC circuit laws (conditions 3 and 4 above) via zero-injection node. Hence, Eq. (3) can be rewritten according to measurement, pseudo-measurement and extended-measurement respectively. The relation between the PMU installation bus and its adjacent bus can be expressed as follows:

$$\begin{bmatrix} \mathbf{Z}_{\mathrm{V}} \\ \mathbf{Z}_{\mathrm{I}} \end{bmatrix} = \mathbf{Hx} = \begin{bmatrix} \mathbf{I} & 0 \\ \mathbf{Y}_{\mathrm{IM}} & \mathbf{Y}_{\mathrm{IN}} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{\mathrm{M}} \\ \mathbf{V}_{\mathrm{M}} \end{bmatrix} \tag{5}$$

where, $\mathbf{V}_{\mathrm{NC}}$ is pseudo-measurement voltage phasor of the neighbor buses.

For zero-injection bus, when all its adjacent voltage phasors are known, it can be regarded as an extended-measurement and