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Solvability and solutions for bus-type extended load flow

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ABSTRACT

In traditional load flow calculation, only three types of buses, *PQ*, *PV*, and $V\theta$, are generally specified. To accommodate the integration of new kinds of power equipment and flexible load flow control facilities, extra bus types are needed in load flow model. The load flow model incorporated with all possible bus types is named as bus-type extended load flow (BELF) for short in this paper. Both Newton–Raphson and decoupled BELF solutions are developed. An important problem for BELF is its solvability, which is carefully studied and topology-based criteria are proposed to ascertain BELF's solvability. Numerical tests are carried out to verify the convergence of the BELF solution and the correctness of the proposed solvability criteria.

1. Introduction

Load flow calculation is a basic function in power system analysis. It is widely used in security analysis, optimal power flow, and some other network analysis applications. Although the study on load flow calculation algorithms have a long history [1–4], some techniques pertaining to special load flows, such as ill-conditioned load flows [5,6] and distribution network load flows [7–10], are still being actively researched. It is noticed that more and more new types of electrical equipment, such as distributed generators (DGs) and flexible AC transmission system (FACTS) components are being installed in power systems. It is improper to model all these new types of electrical equipment as traditional *PQ* or *PV* buses in load flow calculation. On the other hand, introduce extra bus types can make the load flow model to be more confirmed to the performance of new control facilities such as bus voltage control, external network merging and voltage stability analysis.

There are some published papers on modeling new types of equipment by introducing extra bus types, including DGs [11,12], static volt-ampere reactive (VAR) compensators [13], unified power flow controllers [14–16], and other FACTS equipment [17,18]. A current-based modeling technique of the FACTS devices in load flow calculation is proposed in paper [19–21]. On the other hand, for load flow control, there are also several researches include extra bus types to achieve their respective control objective, instead of solving an optimization problem. For example, Refs. [22,23] introduce *PQV* and *P* buses, thus an effective method for bus voltage control has been developed, *PQV* and *P* buses are also introduced for static voltage stability analysis in [24].

Although load flow calculations including extra bus types have been studied for specified purposes, there are still some important problems to be solved for this bus type extended load flow (BELF). This first one is how to formulate a BELF with various bus types and its solution. The second one is how to evaluate the solvability of BELF, because improper combination of bus types may lead to its insolvability.

The main work of this paper is to provide a systematic study on BELF. An important contribution of this paper is analysis the solvability problem for BELF, and very efficient topology-based solvability criteria are given. The content of this paper is organized as follows: The BELF model is presented and all the possible bus types are given in Section 2. Newton–Raphson and fast decoupled solutions are briefly introduced in Section 3. The solvability problem is discussed and topology-based solvability criteria are given in Section 4. In Section 5, extensive numerical tests have been done to verify the convergence of BELF and the solvability criteria.

2. Model

Load flow calculation is based on bus voltage equation, thus for each bus, four electric variables are generally considered: *P*, *Q*, *V*, and θ. Each variable among them may be known or unknown, thus there are 16 possible bus types under all conditions. All the 16 bus types are named according to their known variables, and are listed in Table 1. The traditional bus types are highlighted in grey.

The selection of bus types depends on practical needs. Traditional, *PQ* buses are used to model loads or generators without voltage control ability, and *PV* buses are used to model generators with voltage control ability. By the development of flexible load flow control and new types of electric equipment, more and more extra bus types will be introduced in the load flow model. In fact,

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Table 1
All possible bus types.

P	Q	V	Θ	Bus Type	P	Q	V	θ	Bus Type
√	-	-	-	P	-	-	-	-	0
√	-	-	√	Pθ	-	-	-	√	θ
√	-	√	-	PV	-	-	√	-	V
√	-	√	√	PVθ	-	-	√	√	Vθ
√	√	-	-	PQ	-	√	-	-	Q
√	√	-	√	PQθ	-	√	-	√	Qθ
√	√	√	-	PQV	-	√	√	-	QV
√	√	√	√	PQVθ	-	√	√	√	QVθ

"√" = known, "–" = unknown.

many of them have been used, such as PQV buses, P buses, $PQV\theta$ buses and 0 buses.

The load flow equations considering all possible bus types can be formulated as:

$$\begin{aligned} \Delta P_i &= P_i^{sp} - V_i \sum_{j \in i} V_i (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0 && i \in N_P \\ \Delta Q_i &= Q_i^{sp} - V_i \sum_{j \in i} V_i (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = 0 && i \in N_Q \\ \Delta V_i &= V_i^{sp} - V_i = 0 && i \in N_V \\ \Delta\theta_i &= \theta_i^{sp} - \theta_i = 0 && i \in N_\theta \end{aligned} \tag{1}$$

where N_P, N_Q, N_V, and N_θ are the bus sets, whose respective P, Q, V, and θ are specified. The superscript sp indicates the specified value. In polar coordinates, V and θ are normally used as state variables, and the latter two equations are not necessary. Thus, load flow equations can be written as:

$$\begin{aligned} \Delta P_i &= P_i^{sp} - V_i \sum_{j \in i} V_i (G_{ij} \cos\theta_{ij} + B_{ij} \sin\theta_{ij}) = 0 i \in N_P \\ \Delta Q_i &= Q_i^{sp} - V_i \sum_{j \in i} V_i (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) = 0 i \in N_Q \end{aligned} \tag{2}$$

The existence of each bus type depends on whether its P and Q are specified. In BELF N_P includes all the eight bus types with P specified (P, $P\theta$, PV, $PV\theta$, PQ, $PQ\theta$, PQV and $PQV\theta$) as listed in Table 1 while N_P only include the PQ and PV buses in the conventional load flow. The connotation for N_Q is similar.

3. Solution

3.1. Newton–Raphson solution

BELF can be viewed as an extension of ordinary load flow, and hence its solution can be derived directly from the solution of ordinary load flow, while the extra bus types should be considered properly. The correction equation of Newton–Raphson method is:

$$\boldsymbol{J} \begin{bmatrix} \Delta\boldsymbol{\theta} \\ \Delta\boldsymbol{V} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P} \\ \Delta\boldsymbol{Q} \end{bmatrix} \tag{3}$$

where $\boldsymbol{J}$ is the Jacobian matrix used in the Newton method (which should be a square matrix). It should be noticed that the following necessary condition should be satisfied to make the load flow problem solvable.

Condition 1. $n_P + n_Q + n_V + n_\theta = 2N$, where n_P, n_Q, n_V, and n_θ are the numbers of buses whose P, Q, V, and θ are specified, and N is the total number of buses.

Condition 1 means that the number of known variables in a load flow problem should be equal to that of unknown variables.

The Newton–Raphson correction Eq. (3) can be further expressed as:

$$\begin{bmatrix} \boldsymbol{J}_{P\theta} & \boldsymbol{J}_{PV} \\ \boldsymbol{J}_{Q\theta} & \boldsymbol{J}_{QV} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\theta}_{\overline{N}_\theta} \\ \Delta\boldsymbol{V}_{\overline{N}_V} \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{P}_{N_P} \\ \Delta\boldsymbol{Q}_{N_Q} \end{bmatrix} \tag{4}$$

where

$$\boldsymbol{J}_{P\theta} = \frac{\partial\boldsymbol{P}}{\partial\boldsymbol{\theta}}, \quad \boldsymbol{J}_{PV} = \frac{\partial\boldsymbol{P}}{\partial\boldsymbol{V}}, \quad \boldsymbol{J}_{Q\theta} = \frac{\partial\boldsymbol{Q}}{\partial\boldsymbol{\theta}}, \quad \boldsymbol{J}_{QV} = \frac{\partial\boldsymbol{Q}}{\partial\boldsymbol{V}} \tag{5}$$

$\Delta\boldsymbol{P}_{N_P}$ is the vector of active power unbalance for the buses in N_P, $\Delta\boldsymbol{Q}_{N_Q}$ is the vector of reactive power unbalance for the buses in N_Q. $\Delta\boldsymbol{\theta}_{\overline{N}_\theta}$ is the vector of bus voltage angle correction for the buses beside N_θ, and $\Delta\boldsymbol{V}_{\overline{N}_V}$ is the vector of bus voltage amplitude correction for the buses besides N_V. In (5), $\boldsymbol{J}_{P\theta}$ is a $n_P \times (n - n_\theta)$ matrix:

$$\boldsymbol{J}_{P\theta,(i,j)} = \frac{\partial\Delta\boldsymbol{P}_i}{\partial\boldsymbol{\theta}_j} = \begin{cases} -V_i^2 B_{ii} + V_i \sum_{j \in i} V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) & i = j \\ V_i V_j (G_{ij} \sin\theta_{ij} - B_{ij} \cos\theta_{ij}) & i \neq j \end{cases} \quad i \in N_P, j \notin N_\theta \tag{6}$$

Table 2
Four bus types in P-sub iteration.

Bus type indexes	P	θ	Examples
1	√	–	P, PQ, PV, PQV
2	–	–	0, Q, V, QV
3	√	√	$P\theta$, $PQ\theta$, $PV\theta$, $PQV\theta$
4	–	√	θ, $Q\theta$, $V\theta$, $QV\theta$