

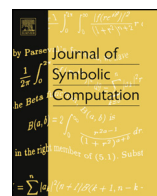


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An algorithm for computing mixed sums of products of Bernoulli polynomials and Euler polynomials

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ABSTRACT

In this paper, by the methods of partial fraction decomposition and generating function, we give an algorithm for computing mixed sums of products of l Bernoulli polynomials and $k-l$ Euler polynomials, which are of the form

$$T_{n,k}^{\lambda}(y; l, k-l) := \sum_{\substack{j_1+\dots+j_k=n \\ j_1, \dots, j_k \geq 0}} \prod_{i=1}^k \lambda_i^{j_i} \binom{n}{j_1, \dots, j_k} \\ \times \prod_{p=1}^l B_{j_p}(x_p) \prod_{q=l+1}^k E_{j_q}(x_q),$$

where $\lambda = (\lambda_1, \dots, \lambda_k)$, and $\lambda_1, \dots, \lambda_k$ are nonzero rational numbers. Moreover, some special sums are presented as examples.

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1. Introduction and preliminary results

The Bernoulli polynomials $B_n(x)$ and Euler polynomials $E_n(x)$ play important roles in various branches of mathematics. They are defined by the generating functions

$$\frac{te^{xt}}{e^t - 1} = \sum_{n=0}^{\infty} B_n(x) \frac{t^n}{n!} \quad \text{and} \quad \frac{2e^{xt}}{e^t + 1} = \sum_{n=0}^{\infty} E_n(x) \frac{t^n}{n!}. \quad (1.1)$$

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They also satisfy the difference equations

$$B_n(x+1) - B_n(x) = nx^{n-1} \quad \text{and} \quad E_n(x+1) + E_n(x) = 2x^n \quad (1.2)$$

and multiplication theorems

$$B_n(mx) = m^{n-1} \sum_{k=0}^{m-1} B_n\left(x + \frac{k}{m}\right) \quad \text{for } m = 1, 2, \dots, \quad (1.3)$$

$$E_n(mx) = \begin{cases} m^n \sum_{k=0}^{m-1} (-1)^k E_n\left(x + \frac{k}{m}\right) & \text{for } m = 1, 3, \dots, \\ -\frac{2}{n+1} m^n \sum_{k=0}^{m-1} (-1)^k B_{n+1}\left(x + \frac{k}{m}\right) & \text{for } m = 2, 4, \dots \end{cases} \quad (1.4)$$

(see Abramowitz and Stegun, 1992, Chapter 23, and Comtet, 1974, Section 1.14). Additionally, the rational numbers $B_n = B_n(0)$ and integers $E_n = 2^n E_n(1/2)$ are called Bernoulli numbers and Euler numbers, respectively.

Many generalizations of these polynomials have been introduced and studied. For example, the higher order Bernoulli polynomials $B_n^{(\alpha)}(x)$ and higher order Euler polynomials $E_n^{(\alpha)}(x)$, each of degree n in x and in α , are defined by the generating functions

$$\left(\frac{t}{e^t - 1}\right)^\alpha e^{xt} = \sum_{n=0}^{\infty} B_n^{(\alpha)}(x) \frac{t^n}{n!} \quad \text{and} \quad \left(\frac{2}{e^t + 1}\right)^\alpha e^{xt} = \sum_{n=0}^{\infty} E_n^{(\alpha)}(x) \frac{t^n}{n!}. \quad (1.5)$$

Clearly, we have $B_n^{(1)}(x) = B_n(x)$ and $E_n^{(1)}(x) = E_n(x)$.

These polynomials and numbers satisfy a large number of identities. In particular, Dilcher (1996) studied the sums of products of arbitrarily many Bernoulli numbers, Bernoulli polynomials, Euler numbers, and Euler polynomials. He found that

$$\begin{aligned} B_n^{(k)}(y) &= \left[\frac{t^n}{n!} \right] \left(\frac{t}{e^t - 1} \right)^k e^{yt} = \sum_{\substack{j_1 + \dots + j_k = n \\ j_1, \dots, j_k \geq 0}} \binom{n}{j_1, \dots, j_k} B_{j_1}(x_1) \cdots B_{j_k}(x_k) \\ &= (-1)^{k-1} k \binom{n}{k} \sum_{j=0}^{k-1} (-1)^j \left\{ \sum_{i=0}^j \binom{k-j-1+i}{i} s(k, k-j+i) y^i \right\} \frac{B_{n-j}(y)}{n-j}, \end{aligned} \quad (1.6)$$

where

$$\binom{n}{j_1, \dots, j_k} = \frac{n!}{j_1! j_2! \cdots j_k!}$$

are the multinomial coefficients, $s(n, k)$ are the Stirling numbers of the first kind, and $y = x_1 + \cdots + x_k$. Based on this identity, we have

$$\begin{aligned} B_n^{(2)}(y) &= -(n-1)B_n(y) + n(y-1)B_{n-1}(y), \\ B_n^{(3)}(y) &= \frac{(n-1)(n-2)}{2} B_n(y) - \frac{n(n-2)}{2} (2y-3)B_{n-1}(y) \\ &\quad + \frac{n(n-1)}{2} (y-1)(y-2)B_{n-2}(y). \end{aligned}$$

Similarly, Dilcher's result on sums of products of Euler polynomials is

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